Scaling Auctions as Insurance: A Case Study in Infrastructure Procurement

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February 2021

Abstract

Most U.S. government spending on highways and bridges is done through “scaling” procurement auctions, in which private construction firms submit unit price bids for each piece of material required to complete a project. Using data on bridge maintenance projects undertaken by the Massachusetts Department of Transportation (MassDOT), we present evidence that firm bidding behavior in this context is consistent with optimal skewing under risk aversion: firms limit their risk exposure by placing lower unit bids on items with greater uncertainty. We estimate bidders’ risk aversion, the risk in each auction, and the distribution of bidders’ private costs. Simulating equilibrium item-level bids under counterfactual settings, we estimate the fraction of project spending that is due to risk and evaluate auction mechanisms under consideration by policymakers. We find that scaling auctions provide substantial savings relative to lump sum auctions and show how our framework can be used to evaluate alternative auction designs.

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†Stanford Graduate School of Business. Email: svass@stanford.edu. This paper was a chapter in our PhD dissertations. We are indebted to our advisors Ariel Pakes, Elie Tamer, Robin Lee, Edward Glaeser, Claudia Goldin, Nathaniel Hendren, Lawrence Katz and Andrei Shleifer for their guidance and support, as well as to Steve Poftak and Jack Moran, Frank Kucharski, Michael McGrath, and Naresh Chetpelly, among other generous public servants at MassDOT. Their support was invaluable. We are also very grateful to Ali Yurukoglu, Andrzei Skrzypacz, Paulo Somaini, Susan Athey and Zi Yang Kang for their many insightful comments and suggestions, and to Cameron Pfliffer, Philip Greengard and Yingbo Ma for the boost that their help gave to our computational capacity.
1 Introduction

Infrastructure investment underlies nearly every part of the American economy and constitutes hundreds of billions of dollars in public spending each year.\(^1\) However, investments are often directed into complex projects that experience unexpected changes. Project uncertainty can be costly to the firms that implement construction—many of whose businesses are centered on public works—and to the government. The extent of firms’ risk exposure depends not only on project design, but also on the mechanism used to allocate contracts. Contracts with lower risk exposure may be more lucrative and thus might invite more competitive bids. As such, risk sharing between firms and the government can play a significant role in the effectiveness of policies meant to reduce taxpayer expenditures.

We study the mechanism by which contracts for construction work are allocated by the Highway and Bridge Division of the Massachusetts Department of Transportation (MassDOT or “the DOT”). Along with 40 other states, MassDOT uses a scaling auction, whereby bidders submit unit price bids for each item in a comprehensive list of tasks and materials required to complete a project. The winning bidder is determined by the lowest sum of unit bids multiplied by item quantity estimates produced by MassDOT project designers. This winner is then paid based on the quantities ultimately used in completing the project.

Scaling auctions thus have several key features. First, they are widespread and common in public infrastructure procurement. Second, they collect bids over units (that is, tasks and materials) that are standardized and comparable across auctions. Third, they implement a partial sharing of risk between the government and private contractors.

To study auction design in this setting, we specify and estimate a model of bidding in scaling auctions with risk averse bidders. Our model characterizes equilibrium bids in two separable steps: an “outer” condition that ensures that a bidder’s score—the weighted sum of unit bids that is used to determine the winner of the auction—is optimally competitive with respect to the opposing bidders, and an “inner” condition that ensures that the unit bids chosen to sum up to the equilibrium score maximize the expected utility of winning. In effect, the “inner” condition constitutes a portfolio optimization problem for bidders. Equilibrium unit bids distribute a bidder’s score across different items, trading off higher expected profits from high bids on items predicted to overrun against higher risk from low bids on other items.

The separability of the “inner” and “outer” problems yields a useful property: given an observation of a bidder’s equilibrium score, her equilibrium unit bids are fully specified by the characterization of her (“inner”) portfolio problem. This allows us to both interpret

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\(^1\) According to the CBO, infrastructure spending accounts for roughly $416B or 2.4\% of GDP annually across federal, state and local levels. Of this, $165B—40\%—is spent on highways and bridges alone.
reduced form patterns of risk-averse bidding behavior and to estimate the primitives that rationalize the unit bids observed in our data, without specifying the data generating process for the scores themselves.

Using a detailed data set obtained through a partnership with MassDOT, we establish the patterns of bidding behavior that motivate our approach. For each auction in our study, we observe the full set of items involved in construction, along with ex-ante estimates and ex-post realizations of the quantity of each item, a DOT estimate of the market unit rate for the item, and the unit price bid that each bidder who participated in the auction submitted. As in prior work on scaling auctions, we show that contractors skew their bids, placing high unit bids on items they believe will overrun the DOT quantity estimates and low unit bids on items they believe will under-run. This suggests that contractors are generally able to predict the direction of ex-post changes to project specifications, and bid so as to increase their ex-post earnings.

Furthermore, our data suggest that contractors are risk averse. As noted in Athey and Levin (2001), risk neutral bidders would be predicted to submit “penny” bids—unit bids of essentially zero—on all but the items that are predicted to overrun by the largest amount. By contrast, the vast majority of unit bids observed in our data are interior (that is, non-extremal), even though no significant penalty for penny bidding has ever been exercised. We show that while contractors bid higher on items predicted to overrun, holding all else fixed, they also bid lower on items that are more uncertain. This suggests that contractors optimize not only with respect to expected profits, but also with respect to the risk that any given expectation will turn out to be wrong.

Risk aversion, combined with inherent project risk, has significant implications for DOT spending, as well as for the efficacy of policies to reduce it. Risk averse bidders internalize a utility cost from uncertainty, and require higher overall bids in order to insure themselves sufficiently to be willing to participate. As such, auction rules that decrease bidders’ exposure to significant losses can be effective toward lowering overall bids, and subsequently lowering DOT payments to the winning bidder.

In order to gauge the level of risk and risk aversion in our data, we estimate a structural model of uncertainty and optimal bidding. In the first stage of our estimation procedure, we use the history of predicted and realized item quantities to fit a model of bidder uncertainty over item quantity realizations. In the second stage, we construct a Generalized Method of Moments (GMM) estimator for bidders’ risk aversion, as well as their private costs, in each project. Our estimator relies only on predictions of optimal unit bids at the auction-bidder-item level, evaluated from each bidder’s portfolio problem under the constraint of her observed score. As such, our identification strategy leverages granular variation in project
composition (e.g., which items are needed, at what market rate, and in what quantities), in addition to more standard project characteristics such as the identity of the designing engineer. As our predictions of optimal bids capture the bidders’ competitive considerations entirely through the score—which is taken as data—our estimation approach does not require strong assumptions about bidders’ beliefs about their opponents, nor does it require exogenous variation in the composition of bidders across auctions.

We use our structural estimates to evaluate not only the cost of uncertainty to the DOT, but also the performance of scaling auctions relative to alternatives used in other procurement settings. Using an independent private values framework, we simulate the equilibrium outcomes under a counterfactual setting in which uncertainty about item quantities is reduced to zero. When bidder predictions are unaffected—the only change is that uncertainty about these predictions is eliminated—we find that DOT spending decreases by 13.5% for the median auction. This suggests that project uncertainty contributes to a substantial risk premium.

However, scaling auctions perform quite well given the level of uncertainty in these projects. The most common alternative type of procurement mechanism is a lump sum auction, in which bidders commit to a total payment for the project at the time of the auction and are liable for all implementation costs afterward. Lump sum auctions require less planning by the DOT, and they incentivize bidders to be economical when they can be. But for projects that are highly standardized and monitored—such the bridge projects in our data—lump sum auctions primarily shift risk from the DOT onto the risk-averse bidders. Seen in this light, scaling auctions provide a powerful lever for the DOT to lower its costs: not only do scaling auctions provide insurance by reimbursing bidders for every item that is ultimately used, but they also allow bidders to hedge their risks through portfolio optimization. In our simulations, we find that moving from the scaling format to a lump sum format would increase DOT spending by 69% for the median auction, although the increase is much lower if ex-post renegotiation is possible.

Given these results, we ask whether scaling auctions can be further improved through a policy that might reasonably be considered by the DOT. The first answer is negative. Although we find a substantial risk premium from eliminating uncertainty holding everything else fixed, a policy to reduce uncertainty—through training or directives, for instance—may not be very effective at reducing costs in practice. Uncertainty in our data is fairly symmetric: under-runs and over-runs both occur frequently, both in quantities and in DOT spending. As such, when we compare the no-uncertainty counterfactual against the status-quo, DOT savings in the median auction are reduced to only 0.28%, with losses at the 25th percentile. This is because eliminating uncertainty gives bidders access to the exact
quantities that will ultimately be used, allowing them to avoid making “mistakes” (from an ex-post perspective) that had benefited the DOT under uncertainty. In some cases, this difference in predictions dominates the elimination of the risk premium. Thus, although we find that reducing uncertainty is beneficial to the DOT on net, it may not hold up to scrutiny when practical considerations are accounted for.

However, an alternative proposal that was raised by MassDOT in 2017—but was ultimately defeated—may be worth considering. We examine a policy in which each item is given a strictly positive unit reservation price. When bidders maximize their portfolios of bids unconstrained, they balance maximizing their expected profits against minimizing their risk. This generally does not result in the risk-minimizing spread. A minimum reservation price forces bidders to bid higher on items that they are willing to be effectively uncompensated for. While this moves bidders away from their optimal portfolio, it also lowers the risk that they undertake. Furthermore, as all bidders are subject to the same constraint, competition entails that the distribution of scores does not change substantially in equilibrium, and so bidders are forced to lower their bids for items that they would strongly overbid otherwise. As a result, the risk premium paid by the DOT decreases endogenously. Considering a unit reservation price of 25% of each item’s market rate, we find that the risk premium decreases by a fifth. A policy of this sort may thus present a cost-effective way to improve on the status quo.

2 Related Literature

Strategic bid skewing in scaling auctions has been documented in various contexts where bidders may be better informed than the auctioneer. Studying US timber auctions, Athey and Levin (2001) first established that positive correlations between (dollar) over-bids and (unit) over-runs in auction data could be interpreted as evidence that bidders are able to predict which components of their bids will overrun. Bajari, Houghton, and Tadelis (2014) made a similar observation in the context of highway paving procurement auctions in California. However, neither paper evaluates the welfare impact of bid-skewing or the underlying uncertainty that causes it.

Bidders who are risk-neutral, such as in the model proposed by Bajari et al. (2014), would be predicted to skew “completely”—that is, bid very high on one component of the project and zero on all the others—unless they face an additional incentive not to do so. Moreover, absent such an incentive, there is no welfare cost to skewing whatsoever: were the government to perfectly predict quantities such that there are no over-runs, the ultimate payment to the winning bidder would be the same. Bajari et al. (2014) accounts for the lack of complete
skewing in their data by imposing a penalty on unit bids that increases in the distance between the bid and the government’s unit cost estimate, as well as the distance between the ex-ante unit quantity estimate and the ex-post realization of that quantity. While this enables Bajari et al. (2014) to structurally estimate average adaptation cost multipliers and calibrate the cost of ex-post renegotiation, the penalty function coefficient is found to be insignificantly different from zero, and no bidder-specific types or counterfactual strategies are estimated.

As in Athey and Levin (2001), our paper argues that the absence of complete skewing is primarily driven by risk aversion. Our model of risk averse bidding predicts that unit bids will be skewed both as a function of bidders’ predictions of ex-post quantities and the amount of uncertainty in each prediction. The heart of our paper rests in the resulting portfolio optimization problem, which determines the spread of unit bids for each score that a bidder submits, and consequently, both the bidder’s private value for winning the auction and the government’s ex-post payment to the bidder if she wins—both of which differ from the score itself.

Our portfolio characterization of bid skewing has several key implications for the analysis of scaling auctions. First, it allows us to construct reduced form correlation tests for risk aversion: much as a positive correlation between over-bids and over-runs is evidence of bidder information, a negative correlation between absolute markups and component-level uncertainty is indicative of risk aversion. Second, it provides a novel channel for identification of bidder and auction-level model parameters. Our identification strategy differs from the canonical approaches of Guerre, Perrigne, and Vuong (2009), Campo, Guerre, Perrigne, and Vuong (2011) and Campo (2012). Like these papers, we require functional form assumptions such as the CARA utility function. However, whereas their approaches rely on the optimality of single-dimensional bids with respect to the probability of beating out other bidders—analogous to the first order condition characterizing the optimal score in our model—our approach uses the optimality of the composition of unit bids to maximize the value of executing a contract conditional on each bidder’s score.

This has important implications for the assumptions about equilibrium play that are required. The Campo and GPV approaches require bids to be interpreted as equilibrium outcomes of an explicit competitive bidding game—whether a symmetric IPV game or an asymmetric affiliated values game. By contrast, our identification approach is agnostic to the competitive conditions under which each bidder’s score is chosen. Subject to comparatively weak conditions that guarantee the separability of the deterministic portfolio-optimization problem from the equilibrium problem of choosing a score for each bidder, our identification strategy is robust to a number of alternative settings, dynamic considerations and
collusion. This does not mean that we are impervious to the non-identification results detailed in Guerre et al. (2009): we still require a parametric (or potentially, semi-parametric) characterization of bidders’ utility and exogenous variation in the distribution of contract values across auctions. However, the particular assumptions needed are different: instead of assumptions about bidders’ beliefs about each other, we use assumptions about bidders’ beliefs about project characteristics. This set of assumptions may be preferable in a highly standardized infrastructure procurement setting such as ours, where historical information is publicly available and bidders are often industry veterans, but where inherent uncertainty about the underlying physical conditions at each project site is high.

Finally, our portfolio approach facilitates counterfactual analyses of alternative auction rules. Because bidders are not paid the score that they compete with—but rather a linear transformation of their unit bids—a prediction of counterfactual scores that does not model the relationship between scores and unit bids would be insufficient to generate predictions for government cost or welfare. Using our model, we evaluate policies to reduce uncertainty regarding item quantities, to pre-commit payments at the time of bidding, and to enforce minimum unit bids on all items.

Our paper contributes to a substantial literature on the efficiency of infrastructure procurement auctions. Closest to us is Luo and Takahashi (2019), a contemporary paper that studies infrastructure procurement by the Florida DOT. Like us, Luo and Takahashi (2019) considers risk averse bidders and compares scaling auctions against lump sum auctions. However, this paper follows a GPV/Campo-style approach for identification and reduces the project components that receive unit bids (which number 67 for the median auction in our dataset) into two aggregates—one with bidder-auction-specific variation and one with a bidder-auction-specific mean—for estimation. As such, while Luo and Takahashi (2019) offers novel evidence of risk aversion and the costliness of lump sum auctions in settings with high uncertainty, we view our analyses as complementary in methodology and contribution.

More generally, our paper builds on a rich literature on scoring auctions. While the theoretical results for risk-neutral bidders in Che (1993) and Asker and Cantillon (2008) do not apply to our model directly, the separability of equilibrium bidding into an ”outer” score-setting stage and an ”inner” portfolio-maximizing stage in our model is closely related to the separability of quality provision and bidding. Our paper also relates to the theoretical literature on optimal mechanism design. While we focus on “practical” mechanisms—ones that do not require knowledge of the bidder type distribution, for instance—it is possible to characterize the theoretically optimal mechanism for our setting by applying the characterization in Maskin and Riley (1984) and Matthews (1987) to our framework.

2See for instance, Lewis and Bajari (2011) and Krasnokutskaya and Seim (2011).
3 Scaling Auctions with MassDOT

Like most other states, Massachusetts manages the construction and maintenance for its highways and bridges through its Department of Transportation. In order to develop a new project, MassDOT engineers assemble a detailed specification of what the project will entail. This includes an itemized list of every task and material (item) that is necessary to complete the project, along with estimates for the quantity with which it will be needed, and a market unit rate for its cost. The itemized list of quantities is then advertised to prospective bidders.

In order to participate in an auction for a given project, a contractor must first be pre-qualified by MassDOT. Pre-qualification entails that the contractor is able to complete the work required, given their staff and equipment. Notably, it generally does not depend on past performance. In order to submit a bid, a contractor posts a unit price for each of the items specified by MassDOT. Since April 2011, all bids have been processed through an online platform, Bid Express, which is also used by 41 other state DOTs. All bids are private until the completion of the auction.

Once an auction is complete, each contractor is given a score, computed by the sum of the product of each item’s estimated quantity and the contractor’s unit-price bid for it. The bidder with the lowest score is then awarded a contract to execute the project in full. In the process of construction, it is common for items to be used in quantities that deviate from MassDOT specifications. All changes, however, must be approved by an on-site MassDOT manager. The winning contractor is ultimately paid the sum of her unit price bid multiplied by the actual quantity of each item used. Unit prices are almost never renegotiated. However, there is a mechanical price adjustment on certain commodities such as steel and gasoline if their market prices fluctuate beyond a predefined threshold (typically 5%).

MassDOT reserves the right to reject bids that are heavily skewed. However, this has never been successfully enforced and most bids violate the condition that should trigger rejection. MassDOT has entertained other proposals to curtail bid skewing, such as a 2017 push to require a minimum unit price on each item. However, this proposal was defeated after bidder protests.

4 Data and Reduced Form Results

Our data come from MassDOT and cover highway and bridge construction and maintenance projects undertaken by the state from 1998 to 2015. We work with projects for which MassDOT has digital records on 1) identities of the winning and losing bidders; 2) bids for

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3 See https://www.mass.gov/service-details/massdot-special-provisions for details.
4 See Section B in the Online Appendix for a detailed discussion.
the winning and losing bidders; and 3) data on the actual quantities used for each item. 2,513 projects meet these criteria, 440 of which are related to bridge work. We focus on bridge projects alone for this paper, as these projects are particularly prone to item quantity adjustments.

All bidders who participate in auctions for these projects are able to see, ex-post, how everyone bid on each item. In addition, all contractors have access to summary statistics on past bids for each item, across time and location. Officially, all interested bidders find out about the specifications and expectations of each project at the same time, when the project is advertised (a short while before it opens up for bidding). Only those contractors who have been pre-qualified at the beginning of the year to do the work required by the project can bid on the project. Thus, contractors do not have a say in project designs, which are furnished either in-house by MassDOT or by an outside consultant.

Once a winning bidder is selected, project management moves under the purview of an engineer working in one of six MassDOT districts around the state. This Project Manager assigns a Resident Engineer to monitor work on a particular project out in the field and to be the first to decide whether to approve or reject under-runs, over-runs, and Extra Work Orders (EWOs). The full approval process of changes to the initial project design involves several layers of review. Under-runs and over-runs, as the DOT defines them and as we will define them here, apply to the items specified in the initial project design and refer to the difference between actual item quantities used and the estimated item quantities. EWOs refer to work done outside of the scope of the initial contract design and are most often negotiated as lump sum payments from the DOT to the contractor. For the purposes of our discussion and analyses, we will focus on under-runs and over-runs in bridge construction and maintenance projects.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Pctl(25)</th>
<th>Median</th>
<th>Pctl(75)</th>
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<td>Project Length (Estimated)</td>
<td>1.53 years</td>
<td>0.89 years</td>
<td>0.88 years</td>
<td>1.48 years</td>
<td>2.01 years</td>
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<tr>
<td>Project Value (DOT Estimate)</td>
<td>$2.72 million</td>
<td>$3.89 million</td>
<td>$981,281</td>
<td>$1.79 million</td>
<td>$3.3 million</td>
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<td># Bidders</td>
<td>6.55</td>
<td>3.04</td>
<td>4</td>
<td>6</td>
<td>9</td>
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<tr>
<td># Types of Items</td>
<td>67.80</td>
<td>36.64</td>
<td>37</td>
<td>67</td>
<td>92</td>
</tr>
<tr>
<td>Net Over-Cost (DOT Quantities)</td>
<td>−$286,245</td>
<td>$2.12 million</td>
<td>−$480,487</td>
<td>−$119,950</td>
<td>$167,933</td>
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<tr>
<td>Net Over-Cost (Ex-Post Quantities)</td>
<td>−$26,990</td>
<td>$1.36 million</td>
<td>−$208,554</td>
<td>$15,653</td>
<td>$275,219</td>
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<tr>
<td>Extra Work Orders</td>
<td>$298,796</td>
<td>$295,173</td>
<td>$78,775</td>
<td>$195,068</td>
<td>$431,188</td>
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Table 1: Summary Statistics

Table 1 provides summary statistics for the bridge projects in our data set. We measure the extent to which MassDOT overpays the projected project cost in two ways. First, we consider the difference between what the DOT ultimately pays the winning bidder and the DOT’s initial estimate of what it will pay at the conclusion of the auction. Summary
statistics for this measure are presented in the “Net Over-Cost (DOT Quantities)” row of Table 1.

While it seems as though the DOT is saving money on net, this is a misrepresentation of the costs of bid skewing. The initial estimate—which uses the DOT’s ex-ante quantity estimates and corresponds to the winning bidder’s score in our model—is not necessarily representative of the payments that the bidder expects upon winning. Sophisticated bidders anticipate changes from the initial DOT estimates, and bid accordingly to maximize their ex-post payments. As such, a more appropriate metric is to compare the amount that was ultimately spent in each project against the dot product of the DOT’s unit cost estimates and the actual quantities used. This is presented in the “Net Over-Cost (Ex-Post Quantities)” row of Table 1. The median over-payment by this metric is about $15,000, but the 25th and 75th percentiles are about -$210,000 and $275,000. Figure 1 shows the spread of over-payment across projects. As we will show in our counterfactual section, the distribution of over-payment corresponds to the potential savings from the elimination of risk.

![Figure 1: Net Over-Cost (Ex-Post Quantities) Across Bridge Projects](image)

**Bidder Characteristics**  There are 2,883 unique project-bidder pairs (i.e., total bids submitted) across the 440 projects that were auctioned off. There are 116 unique firms that participate, albeit to different degrees. We divide them into two groups: ‘common’ firms, which participate in at least 30 auctions within our data set, and ‘rare firms’, which participate in fewer than 30 auctions. We retain individual identifiers for each of the 24 common firms, but group the 92 rare firms together for purposes of estimation. Common firms constitute 2,263 (78%) of total bids submitted and 351 (80%) of auction victories.

Although there is little publicly available financial information about them, the firms in our data are by and large relatively small, private, family-owned businesses. Table 2 presents summary statistics of the two firm groups. The mean (median) common firm submitted bids to 94.29 (63) auctions and won 14.62 (10) of them. The mean total bid (or score) is about $2.8
<table>
<thead>
<tr>
<th></th>
<th>Common Firm</th>
<th>Rare Firm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Firms</td>
<td>24</td>
<td>92</td>
</tr>
<tr>
<td>Total Number of Bids Submitted</td>
<td>2263</td>
<td>620</td>
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<tr>
<td>Mean Number of Bids Submitted Per Firm</td>
<td>94.29</td>
<td>6.74</td>
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<tr>
<td>Median Number of Bids Submitted Per Firm</td>
<td>63.0</td>
<td>2.5</td>
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<tr>
<td>Total Number of Wins</td>
<td>351</td>
<td>89</td>
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<tr>
<td>Mean Number of Wins Per Firm</td>
<td>14.62</td>
<td>0.97</td>
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<tr>
<td>Median Number of Wins Per Firm</td>
<td>10</td>
<td>0</td>
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<tr>
<td>Mean Bid Submitted</td>
<td>$2,774,941</td>
<td>$4,535,310</td>
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<tr>
<td>Mean Ex-Post Cost of Bid</td>
<td>$2,608,921</td>
<td>$4,159,949</td>
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<tr>
<td>Mean Ex-Post Over-run of Bid</td>
<td>9.7%</td>
<td>21.97%</td>
</tr>
<tr>
<td>Percent of Bids on Projects in the Same District</td>
<td>28.19%</td>
<td>15.95%</td>
</tr>
<tr>
<td>Percent of Bids by Revenue Dominant Firms</td>
<td>51.67%</td>
<td>11.80%</td>
</tr>
<tr>
<td>Mean Specialization</td>
<td>24.44</td>
<td>2.51</td>
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<tr>
<td>Mean Capacity</td>
<td>10.38</td>
<td>2.75</td>
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<tr>
<td>Mean Utilization Ratio</td>
<td>53.05</td>
<td>25.50</td>
</tr>
</tbody>
</table>

Table 2: Comparison of Firms Participating in <30 vs 30+ Auctions

million, while the mean ex-post DOT cost implied by the firm’s unit bids is $2.6 million. The mean ex-post cost over-run is 9.73%. By contrast, the mean (median) rare firm submitted bids to 6.74 (2.5) auctions and won 0.97 (0) of them. The mean total bid and ex-post scores are quite a bit larger for rare firms—$4.5 million and $4.2 million, respectively. This is reflected in a substantially larger ex-post over-run: 21.97% on average.

In addition to the firm’s identity, there are a number of factors which may influence its competitiveness in a given auction. While we do not consider a structural interpretation for these factors in our model, we treat them as characteristics that help explain heterogeneity in cost types across firms and auctions. One such factor is the firm’s distance from the worksite. Although we do not observe precise locations for each project, we observe which of the six geographic districts under MassDOT jurisdiction each project belongs to. We then geocode the headquarters of each firm by district, and compare districts for each project-bidder pair. Among common firms, 28.19% of bids were on projects that were located in the same district as the bidding firm’s headquarters. By contrast, only 15.95% of bids among rare firms were in matching districts.

Another factor is specialization or experience with a particular type of project. We calculate the specialization of a project-bidder pair as the share of auctions of the same project type that the bidding firm has placed a bid on within our dataset. Our data involve three
distinct project types, according to DOT taxonomy: Bridge Reconstruction/Rehabilitation, Bridge Replacement, and Structures Maintenance. The mean specialization of a common firm is 24.44%, while the mean specialization of a rare firm is 2.51%. As projects have varying sizes, we compute a measure of specialization in terms of project revenue as well. We define a revenue-dominant firm (within a project-type) as a firm that has been awarded more than 1% of the total money spent by the DOT across projects of that project type. Among common firms, 51.67% of bids submitted were by firms that were revenue dominant in the relevant project type; among rare firms, the proportion of bids by revenue dominant firms is 11.8%.

A third factor of competitiveness is each firm’s capacity—the maximum number of DOT projects that the firm has ever had open while bidding on another project—and a fourth factor is its utilization—the share of the firm’s capacity that is filled when it is bidding on the current project. We measure capacity and utilization with respect to all MassDOT projects recorded in our data—not just bridge projects. The mean capacity is 10.38 projects among common firms and 2.75 projects among rare firms. This suggests that rare firms generally have less business with the DOT, either because they are smaller in size, or because the DOT constitutes a smaller portion of their operations. The mean utilization ratio, however, is 53.05% for common firms and 25.5% for rare firms. This suggests that firms in our data are likely to have ongoing business with the DOT at the time of bidding and are likely to have spare capacity during adjacent auctions that they did not participate in. While we do not model the dynamic considerations of capacity constraints directly, we find our measure of capacity to be a useful metric of the extent of a firm’s dealings with the DOT, as well as its size.

**Quantity Estimates and Uncertainty** As we discuss in Section 8, scaling auctions mitigate DOT costs by enabling risk-averse bidders to insure themselves against uncertainty about the item quantities that will ultimately be used for each project. The welfare benefit is particularly strong if the uncertainty regarding ex-post quantities varies across items within a project, and especially so if there are a few items that have particularly high variance. When this is the case, bidders in a scaling auction can greatly reduce the risk that they face by placing minimal bids on the uncertain items (and higher bids on more predictable items).

Our data set includes records of 2,985 unique items, as per MassDOT’s internal taxonomy. Spread across 440 projects, these items constitute 29,834 unique item-project pairs. Of the 2,985 unique items, 50% appear in only one project. The 75th, 90th, and 95th percentiles of unique items by number of appearances in our data are 4, 16, and 45 auctions, respectively.

For each item, in every auction, we observe the quantity with which the DOT predicted it
would be used at the time of the auction—\( q^e_t \) in our model—the quantity with which the item was ultimately used—\( q^a_t \)—and a DOT engineers’ estimate of the market rate for the unit cost of the item. The DOT quantities are typically inaccurate: 76.7% of item observations in our data had ex-post quantities that deviated from the DOT estimates.

Figure 2a presents a histogram of the percent quantity over-run across item observations. The percent quantity over-run is defined as the difference of the ex-post quantity of an item observation and its DOT quantity estimates, normalized by the DOT estimate: \( \frac{q^e_t - q^a_t}{q^e_t} \). In addition to the 23.3% item-project observations in which quantity over-runs are 0%, another 18% involve items that are not used at all (so that the over-run is equal to -100%). The remaining over-runs are distributed more or less symmetrically around 0%.

The ex-post deviations from DOT quantity estimates result from a number of different mechanisms. Some deviations arise from standard procedures. For instance, as ex-ante DOT estimates are used for budgeting purposes, there may be reason for adjusting the quantities of certain items after the design stage. One example is concrete, which is heavily used, has quantities that are difficult to predict precisely, and often overruns in our data. It is also common for the DOT to include certain items that are unlikely to be used at all—just in case—in order to support its policy of avoiding ex-post renegotiation. Prominent examples of such items include flashing arrows and illumination for night work. While mechanisms of this sort are largely systemic, there remains a substantial amount of variation in ex-post quantities simply due to the inherent uncertainty of construction. A large fraction of Massachusetts bridges are structurally deficient, making it difficult to ascertain the exact severity of their condition prior to construction. From our conversations with DOT engineers, these mechanisms are known symmetrically by all of the bidders. Motivated by profit, bidders are also generally thought to have better institutional knowledge and quality predictive software than the DOT.

We document, furthermore, that quantity over-runs vary across observations of the same item in different auctions. Figure 2b plots the mean percent quantity over-run for each unique item with at least 2 observations against its standard deviation. While a few items have standard deviations close to 0, the majority of items have over-run standard deviations that are as large or larger than the absolute value of their means. That is, the percent over-run of the majority of unique items varies substantially across observations. While this is a coarse approximation of the uncertainty that bidders face with regard to each item—it does not take item or project characteristics into account, for example—it is suggestive of the scope of risk in each auction.

**Reduced Form Evidence for Risk Averse Bid Skewing**  As in Athey and Levin (2001) and Bajari et al. (2014), the bids in our data are consistent with a model of similarly in-
formed bidders who bid strategically to maximize expected utility. In Figure 3a, we plot the relationship between quantity over-runs and the percent by which each item was overbid above the blue book cost estimate. We do this for both the winning bidder and the second place bidder. The binscatter is residualized. In order to obtain it, we first regress percent over-bid on a range of controls and obtain residuals. We then regress percent over-run on the same controls and obtain residuals. Finally, to obtain the slope, we regress the residuals from the first regression on the residuals from the second. Controls include the DOT estimate of total project cost, the initially stated project length in days, and the number of participating bidders, as well as fixed effects for: item IDs, the year in which the project was opened for bidding, the project type, resident engineer, project manager, and project designer. Specifications that exclude item fixed effects or include an array of additional controls produce very similar slopes. We use a similar procedure for all residualized binscatters in this section.

As Figure 3a demonstrates, there is a significant positive relationship between percent quantity over-runs and percent over-bids by the winning bidder. A 1% increase in quantity over-runs corresponds to a 0.086% increase in over-bids on average. Higher bids on over-

\[ \text{Percent over-bid of an item is defined as } \frac{b_t - c_t}{c_t} \times 100, \text{ where } b_t \text{ is the bid on item } t \text{ and } c_t \text{ is the DOT market rate estimate of item } t. \] The percent quantity over-run is similarly defined as \[ \frac{q^a_t - q^e_t}{q^e_t} \times 100, \] where \( q^a_t \) is the amount of item \( t \) that was ultimately used and \( q^e_t \) is the DOT quantity estimate for item \( t \) that is used to calculate bidder scores.

\[ \text{For each graph, we truncate observations at the top and bottom 1%. This is done for the purposes of clarity as outliers can distort the visibility of the general trends.} \]
running items correspond to higher earnings ex-post. Thus, as higher bids correspond to items that overran in our data, we conclude that the winning bidder is able to correctly predict which items will overrun the DOT estimates on averages, and to skew strategically accordingly.

Furthermore, Figure 3a shows that losing bidders skew their bids in a similar way to winning bidders. With the exception of a few outlying points, the relationship between over-bids and over-runs is very similar between the top two bidders: they both overbid on items that wound up overrunning on average. This suggests that the winning and second place bidder are similarly able to predict over-runs. In Appendix G, we show that this relationship for the second-place bidder is even stronger when we restrict our comparison to projects in which the first two bidders submit similar total scores and thus have similar private costs of production.

(a) Residualized binscatter of item-level percent over-bid by the rank 1 (winning) and rank 2 bidder, against percent quantity over-run. (b) Binscatter of item-level percent over-bids by the rank 2 bidder against the rank 1 (winning) bidder.

Figure 3

While our data suggest that bidders do engage in bid skewing, there is no evidence of total bid skewing, in which a few items are given very high unit bids and the rest are given “penny bids”. The average number of unit bids worth $0.10 or less by the winning bidder is 0.51—or 0.7% of the items in the auction. The average number of unit bids worth $0.50, $1.00, and $10.00, respectively, is 1.68, 2.85 and 13.91, corresponding to 2.8%, 4.73%, and 23.29% of the items in the auction. This observation is consistent with previous studies of bidding in scaling auctions. Athey and Levin (2001) argue that the interior bids observed in their data are suggestive of risk aversion among the bidders. While they acknowledge that other forces, such as fear of regulatory rebuke, may provide an alternative explanation for the lack of total bid skewing, they note that risk avoidance was the primary explanation.
given to them in interviews with professionals.

In addition to interior bids, risk aversion has several testable theoretical implications. Risk averse bidders balance the incentive to bid high on items that are projected to overrun with an incentive to bid closer to cost on items that are uncertain. As our model in Section 5 shows, bidders with higher costs and higher scores face larger amounts of risk from extremal bids. As such, they are less willing to skew strongly or bid far below cost on items predicted to under-run.\(^7\)

This observation is consistent with the pattern demonstrated in Figure 3a. Here, the second place bidder—who submitted a higher overall score by definition—generally exhibits less severe skewing: a 1% increase in quantity over-runs corresponds to only a 0.019% increase in over-bids on average. Figure 3b, which plots a residualized binscatter of the second place bidder’s unit bid for each item against the winning bidder’s unit bid for the same item, shows a similar pattern.\(^8\) While the direction of skewing corresponds strongly between the top two bidders—a higher over-bid by the winning bidder corresponds to a higher over-bid by the second place bidder as well—the second place bidder’s skewing is more subdued.

The bids in our data also exhibit more direct evidence of risk aversion. We would expect risk averse bidders to bid lower markups on items that—everything else held fixed—have higher uncertainty. While we do not see observations of the same item in the same context with identifiably different uncertainty, we present the following suggestive evidence that such behavior is occurring.

In Figures 4a and 4b, we plot the relationship between the unit bid for each item in each auction by the winning bidder, and an estimate of the level of uncertainty regarding the ex-post quantity of that item (in the context of the particular auction). To calculate the level of uncertainty for each item, we use the results of our first stage estimation, discussed in Section 6. For every item, in every auction, our first stage gives us an estimate of the variance of the error for the best prediction of what the ex-post quantity of that item would be, given information available at the time of bidding.

In Figure 4a, we plot a residualized binscatter of the winning bidder’s absolute percent over-bid on each item against the item’s standard deviation—the square root of the estimated prediction variance. This captures the uncertainty of each item quantity prediction across auctions in which it may appear with different DOT expectations and project compositions and characteristics. The relationship is negative, suggesting that holding all else fixed, bidders bid closer to cost on items with higher variance and thus limit their risk exposure.

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\(^7\)See Supplemental Appendix A.2 for a worked out example and intuition.

\(^8\)Note that the percent over-bids in Figure 3b appear to be substantially larger than those in Figure 3a. This is because while large over-bids occur in the data, they are relatively rare and so are averaged down in the percent quantity over-run binning in Figure 3a.
Note, however, that this analysis does not directly account for the trade-off between quantity over-runs and uncertainty. As in Equation (5), a bidder’s certainty equivalent increases in the predicted quantity of each item, but decreases in the item’s quantity variance. To account for this trade-off, we consider the following alternative metric for bidding high on an item:

\[
\% \Delta \text{ Revenue Contribution from } t = \frac{\sum_p b_p q_p^t - \sum_p c_t q_p^t}{\sum_p c_t q_p^t} \times 100
\]

This is the percentage difference in the proportion of the total revenue earned by the winning bidder from item \(t\), and the proportion of the DOT’s initial cost estimate that item \(t\) constituted. We take the percent difference between the item’s revenue contribution to the bidder and its cost contribution to the DOT’s total estimate in order to normalize across items that inherently play a bigger or smaller role in a project’s total cost. In Figure 4b, we plot the residualized binscatter of the \(\% \Delta \text{ Revenue Contribution} due to each item against the item’s quantity standard deviation. The negative relationship here is particularly pronounced, providing further evidence that bidders allocate proportionally less weight in their expected revenue to items with high variance. Our model of risk averse bidding predicts exactly this kind of relationship.
5 A Structural Model for Bidding With Risk Aversion

In this section, we present the theoretical framework underlying our empirical exercise. We first present our baseline model, in which bidders competing in an auction share a common CARA utility function. We then discuss extensions to CRRA utility and asymmetric risk aversion types.

5.1 The Baseline Model

A procurement auction consists of \( N \) qualified bidders competing for a contract to complete a single construction project. Each project is characterized by \( T \) items, each of which is ultimately needed in a different quantity. Prior to bidding, bidders observe a DOT estimate \( q^e_t \) for each item \( t \)'s quantity, as well as an additional noisy public signal \( q^b_t \). Although we do not model this explicitly until Section 6, the public signal should be thought of as a refinement of \( q^e_t \) that incorporates further public information, such as the identity of the designing engineer and historical trends for similar projects. Upon completion of construction, the actual quantity \( q^a_t \) of each item is realized, independently of which bidder won the auction and at what price. To summarize, there are three kinds of quantity objects:

- \( q^e = \{q^e_1, \ldots, q^e_T\} \): DOT estimates based on underlying conditions at the project site
- \( q^b = \{q^b_1, \ldots, q^b_T\} \): Common refined estimates based on public information
- \( q^a = \{q^a_1, \ldots, q^a_T\} \): Actual quantities, realized ex-post independently of the auction

In order to participate in the auction, each bidder \( i \) must submit a unit bid \( b_{i,t} \) for every item \( t \) involved in the auction. Bids are simultaneous and sealed until the conclusion of the auction. To determine a winner, each bidder \( i \) is given a score based on her unit bids and the DOT quantity estimates: \( s_i = \sum_{t=1}^{T} b_{i,t} q^e_t \). The bidder with the lowest score wins the contract and executes the project in full. Once the project is complete, the winning bidder is paid her unit bid \( b_{i,t} \) for each item \( t \) multiplied by the actual quantity of \( t \) that was needed, \( q^a_t \).

Bidder Types The winner of a procurement auction is responsible for securing all of the items required to complete construction. The majority of these items—such as concrete and traffic cones—are standard, competitive goods that have a commonly-known market unit cost \( c_t \) at the time of the auction. However, bidders differ in their labor, storage and transportation costs across different projects. To capture this, we assume that bidders differ along a single-dimensional efficiency multiplier \( \alpha \). That is, for every item \( t \) required for a
project, bidder $i$ faces a unit cost of $\alpha_i c_t$, where $\alpha_i$ is the bidder’s efficiency type. Each bidder privately observes her efficiency type prior to bidding. However, it is common knowledge that efficiency types are drawn independently from a common, publicly known distribution over a compact subset $[\underline{\alpha}, \overline{\alpha}]$ of $\mathbb{R}_+$.  

**Uncertainty and Risk Aversion**  
Bidder expectations for the quantities with which different items will be needed are noisy to different degrees. For tractability, we assume that the bidders’ public signal for each item $t$ in our baseline specification is normally distributed around the actual quantity of $t$, with an item-specific variance parameter:

$$q^b_t = q^a_t + \varepsilon_t, \text{ where } \varepsilon_t \sim \mathcal{N}(0, \sigma_t^2). \quad (1)$$

In addition, we assume that bidders are risk averse with a standard CARA utility function over their earnings from the project and a common constant coefficient of absolute risk aversion $\gamma$:

$$u(\pi) = 1 - \exp(-\gamma \pi). \quad (2)$$

**Bidder Payoffs**  
If a bidder loses the auction, she does not pay or earn anything regardless of her bid. If bidder $i$ wins the auction with bid vector $b_i$, she profits the difference between her unit bid and her unit cost for each item, multiplied by the quantity with which the item is ultimately used: $\sum_t q^a_t \cdot (b_{i,t} - \alpha_i c_t)$. As the realization of $q^a$ is unknown at the time of bidding, bidders face two sources of uncertainty in bidding: uncertainty about their probability of winning and uncertainty about the profits they would earn upon winning. Thus, bidder $i$’s expected utility from participating in the auction is given by:

$$\left(1 - \mathbb{E}_{q^a} \left[ \exp \left( -\gamma \sum_{t=1}^T q^a_t \cdot (b_{i,t} - \alpha_i c_t) \right) \right] \right) \times \left( \Pr \left\{ b_i \cdot q^e < s_j \text{ for all } j \neq i \right\} \right).$$

This is bidder $i$’s expected utility from the profit she would earn if she were to win the auction, multiplied by the probability that her score—at the chosen unit bids—will be the lowest one offered, so that she will win. Substituting the bidders’ Gaussian signal from
Equation (1) and taking the expectation, bidder $i$’s expected utility is given by:

$$
\left(1 - \exp\left(-\gamma \sum_{t=1}^{T} q_{i,t}^{b}(b_{i,t} - \alpha_{i}c_{t}) - \frac{\gamma \sigma_{t}^{2}}{2} (b_{i,t} - \alpha_{i}c_{t})^{2}\right)\right) \\
\times \left(\Pr\left\{b_{i} \cdot q^{e} < s_{j} \text{ for all } j \neq i\right\}\right).
$$

Separability of the Bidder’s Problem  Notably, bidder $i$’s expected utility from participating in the auction is separable in the following two ways. (1) The probability of winning is entirely determined by the score $s_{i} = b_{i} \cdot q^{e}$ and the distribution of opponent bids. Thus, any selection of unit bids that sums to the same score yields the same probability of winning. (2) The expected utility of winning for bidder $i$ depends only on the selection of unit bids submitted by $i$, and is independent of any other bidder’s bids.

This separability property implies that a bidder’s score is payoff-sufficient for her choice of unit bids: in any equilibrium, the vector of unit bids submitted by each bidder must maximize the bidder’s expected utility from winning, conditional on the constraint that the bids sum to the bidder’s equilibrium score. This maximization—which we call the bidder’s portfolio problem—is a deterministic unilateral optimization problem: it does not depend on the bidder’s beliefs about her competition. Instead, all equilibrium considerations are channeled through the choice of the bidder’s equilibrium score, which disciplines the portfolio problem through a linear constraint on the unit bids allowed.

Characterizing Equilibrium  The pure-strategy Bayes Nash Equilibrium of our baseline auction game is characterized by the solution to the following two-stage problem. In the first stage, each bidder $i$ chooses a score $s^{*}(\alpha_{i})$ based on her efficiency type $\alpha_{i}$. This determines the bidder’s probability of winning and constrains the second stage of her bidding strategy. In the second stage, bidder $i$ chooses a vector of unit bids $b_{i}$ that solve her portfolio problem, subject to the constraint that $b_{i} \cdot q^{e} = s^{*}(\alpha_{i})$.

In order for the bids to constitute an equilibrium, $b_{i}$ must maximize bidder $i$’s expected utility conditional on winning—Expression (3)—subject to the score constraint. This optimization problem is strictly convex, and so it has a unique global maximum for any given score. Furthermore, applying a monotone transformation to Expression (3), this problem reduces to a constrained quadratic program, similar to those studied in standard asset pricing.
texts.\textsuperscript{9} 

\[ b^*_t(s) = \max_{b_t} \left[ \gamma \sum_{t=1}^{T} q_t^b (b_{i,t} - \alpha_i c_t) - \gamma \sigma_t^2 \frac{(b_{i,t} - \alpha_i c_t)^2}{2} \right] \]  \hspace{1cm} (5) 

s.t. \ \sum_{t=1}^{T} b_{i,t} q_t^e = s \text{ and } b_{i,t} \geq 0 \text{ for all } t.

As unit bids cannot be negative, the portfolio problem in Equation (5) does not have a closed form solution, and must be solved numerically. However, the optimal unit bid for each item receiving positive weight in the portfolio has the following form:

\[ b^*_{i,t}(s) = \alpha_i c_t + \frac{q_t^b}{\gamma} + \frac{q_t^e}{\sigma_t^2} \sum_{r:b^*_r(s) > 0} \left( \frac{q_r^e}{\sigma_r^2} \gamma \right) \left( s - \sum_{r:b^*_r(s) > 0} q_r^e \left[ \alpha_r c_r + \frac{q_r^b}{\sigma_r^2} \gamma \right] \right). \]  \hspace{1cm} (6)

Note that the optimal bid for each item is a function, not only of that item’s own unit cost and expected quantity to variance ratio, but also of the costs, expectations and variances of the other items receiving positive weight in the optimal portfolio—as well as the bidder’s score. As such, variation in the composition of project needs and uncertainty would induce variation in unit bids even if the competitive structure (e.g., the participating bidders and their private costs) were the same.

Furthermore, both unit bids and the value of the portfolio problem are increasing in \( s \), holding all else fixed. Thus, the portfolio problem provides a unique monotonic mapping from scores to the value that each bidder would receive from winning the auction with each score. Similarly, holding all else fixed, the value of the portfolio problem is decreasing in \( \alpha_i \).\textsuperscript{10} Thus, by Maskin and Riley (2000), there is a unique symmetric equilibrium in monotone strategies mapping efficiency types to scores (and consequently optimal unit bids). For the purpose of estimation, we focus primarily on the solution to the portfolio problem, subject to the score that is observed from each bidder in the data. We defer further discussion of equilibrium construction to Section 8.

\textsuperscript{9}See Campbell (2017) for a survey.

\textsuperscript{10}To ensure the uniqueness of the solution, we define a deterministic tie-breaking rule for solving the quadratic program. Monotonicity of the value of winning the auction in \( s \) and \( \alpha \) also requires scores to be below a zero-profit boundary condition for the least competitive (highest \( \alpha \)) type, as otherwise bids and scores are undefined.
5.2 Extensions

The key observation in Section 5.1 is that using the equilibrium score as a constraint on the portfolio problem is sufficient for characterizing equilibrium unit bids as a function of common observables. Our baseline embeds this observation in a simple model of bidder competition, in which bidders vary along a single-dimensional type. However, the observation holds much more generally. It only requires that the selection of unit bids itself—conditional on the score constraint—impacts bidders’ payoffs through the profits from executing the project alone.

This rules out complex collusive strategies where bidders may use unit bids to send signals to each other, or games in which bidding high on some item induces not only moral hazard but also economies of scale down the line. However, the observation is consistent with a broad range of other modeling assumptions. First, it is consistent with alternative models of risk preference. In Appendix C.1 we replicate our main results for the CRRA case. Second, it is consistent with models of multi-dimensional types such as heterogeneous risk-aversion coefficients and item-specific cost types. While characterizing equilibria in multi-dimensional auctions is notoriously intractable, we discuss how our main results extend to a model with asymmetric risk-aversion types in Appendix C.2.

Furthermore, the sufficiency observation enables estimation for much more complex models of equilibrium considerations. For instance, while the possibility of extra work orders may incentivize a bidder to bid more aggressively, the extra work order is not itself given a unit bid. Thus, the extra incentive would result in a lower score and, consequently, unit bids that are optimal given that score. Similarly, if capacity constraints or other dynamic considerations affected the medium-run value of winning a given auction, the resulting equilibrium score would capture the additional incentives, but the portfolio problem conditional on the score observed would remain the same. This even holds for models of collusion: if bidders take turns submitting the lowest score to win, for instance, the portfolio problem solution conditional on the collusive score of each bidder would remain valid for identification so long as bidders are able to extract the maximum value of winning under the collusive agreement.

6 Econometric Model

We now present a two-step estimation procedure to estimate the primitives of our baseline model. We split our parameters into two categories: (1) statistical/historical parameters, which we estimate in the first stage and (2) economic parameters, which we estimate in the second stage. The first set of parameters characterizes the bidders’ beliefs over the distribution of actual quantities $q^a$. The estimation procedure for this stage employs the full history of auctions in our data to build a statistical model of bidder expectations using
publicly available project and item characteristics. However, it does not take into account information on bids or bidders in any auction. By contrast, the second stage estimates the bidders’ efficiency types $\alpha$ in each auction, as well as an auction-specific CARA coefficient $\gamma$. For this stage, we take the first stage estimates as fixed and construct moments for GMM estimation based on idiosyncratic deviations between observed unit bids and optimal unit bids given by Equation (6).

Stage 1: Estimating the Distribution of the Quantity Signals  
In the model presented in Section 5, we did not take a stance on what the signals in Equation (1) are based on. The reason for this was to emphasize the flexibility of our model with respect to possible signal structures: the only required assumption is that conditional on all of the information held at the time of bidding, the bidders’ common belief of the posterior distribution of each $q^a_t$ can be approximated by a normal distribution with a commonly known mean and variance. This allows for correlations between items, as well as complicated forms of correlation with $q^e_t$.

For the purpose of estimation, however, we make an additional assumption. Denoting an auction by $n$, we assume that the posterior distribution of each $q^a_{t,n}$ is given by a statistical model that conditions on $q^e_{t,n}$, item characteristics (e.g. the item’s type classification), observable project characteristics (e.g. the project’s location, project manager, designer, etc.), and the history of DOT projects. In particular, we model the realization of the actual quantity of item $t$ in auction $n$ as:

$$q^a_{t,n} = \hat{q}^b_{t,n} + \eta_{t,n}, \text{ where } \eta_{t,n} \sim \mathcal{N}(0, \hat{\sigma}^2_{t,n})$$

(7)

such that

$$\hat{q}^b_{t,n} = \beta_{0,q} q^e_{t,n} + \tilde{\beta}_q X_{t,n} \text{ and } \hat{\sigma}_{t,n} = \exp(\beta_{0,\sigma} q^e_{t,n} + \tilde{\beta}_\sigma X_{t,n}).$$

(8)

Here, $\hat{q}_{t,n}$ is the posterior mean of $q^a_{t,n}$ and $\hat{\sigma}_{t,n}$ is the square root of its posterior variance—linear and log-linear functions of the DOT estimate for item $t$’s quantity $q^e_{t,n}$ and a matrix of item-project characteristics $X_{t,n}$. We estimate this model with Hamiltonian Monte Carlo and use the posterior mode as a point estimate for the second stage of estimation.\(^\text{11}\) We demonstrate the goodness of fit in Section 7.

Note that our model allows for correlations between item means ($\hat{q}^b_{t,n}$) and variances ($\hat{\sigma}^2_{t,n}$) through observables, but assumes that deviations ($\eta_{t,n}$) from the means are independent across items within an auction. This is not a binding constraint from a theoretical perspective: in principle, our approach could accommodate correlations across $\eta_{t,n}$ as well.

\(^\text{11}\)We use Hamiltonian Monte Carlo as an efficient implementation of a likelihood method, optimized for a GLM. We discuss the flexibility of our approach to alternative models for the first stage in Appendix D.1.
However, in contrast to the asset pricing literature, each observation of a “portfolio” in our data is composed of a different basket of items. As such, a consistent correlation is in general difficult to identify and estimate.

Our model of bidder quantity signals can be thought of in several ways. It can be interpreted as an additional component of the structural model: the bidders use our method as a statistical estimation procedure to assess the likelihood of item quantities prior to bidding. The DOT quantities, item and project characteristics are indeed all publicly known at the time of bidding, as are historical records of DOT projections and ex-post quantities. Furthermore, there is a mature industry of software for procurement bid management that touts sophisticated estimation of project input quantities and costs. It is thus likely that firms use off-the-shelf tools to forecast project needs. Alternatively, this assumption could be thought of as the econometrician’s model of each signal mean $q_t^b$ and variance $\sigma_t^2$.

**Stage 2: Estimating Efficiency Types and Risk Aversion** Our dataset contains a unit bid for every item, submitted by every participating bidder in every auction that we see. In particular, we have three main sources of heterogeneity: (1) bids submitted by different bidders in the same auction; (2) bids submitted across the different items by each bidder in an auction (3) bids submitted for the same item across different bidders and different auctions. Every auction carries a different set of project characteristics, a different composition of items to be bid, different quantity expectations and variances, and a different set of participating bidders at different states of activity. The optimal bid for every bidder and item depends on the full combination of these features in each auction—not only how competitive the auction is (which affects the unit bids through the bidder’s score), but also which items are required, which are over or underestimated, and which are more or less uncertain given the project observables. Our key identifying assumption is that the optimal unit bid for each item is observed by the econometrician with a measurement error that is exogenous to these features.

**Assumption 1.** Let $b_{t,i,n}^d$ denote the unit bid for item $t$ submitted by bidder $i$ in auction $n$, as observed in our data. Each observed unit bid is equal to the optimal bid $b_{t,i,n}^*$, subject to an IID, mean-zero measurement error $\nu_{t,i,n}$:

$$b_{t,i,n}^d = b_{t,i,n}^* + \nu_{t,i,n}$$

where $E[\nu_{t,i,n}] = 0$ and $\nu_{t,i,n} \perp X_{t,n}, X_{i,n}, X_n$.

Assumption 1 states that each unit bid observed in our data is given by the optimal bid formula from our model—Equation (6) evaluated at the observed score for each bidder—subject to an idiosyncratic error that is independent across draws, and orthogonal to auction-item,
auction-bidder and auction-wide characteristics. Such an error might come about because of rounding/smudging in the translation between the bidder’s optimal bidding choice and the record available to the DOT (and consequently, to the econometrician). One might alternatively frame this error as an optimization error: the optimal choice of bids is a numerical solution to a constrained quadratic program that may not produce numbers that are convenient to report in currency.

In order to predict $b_{t,i,n}^*$ as a function of observables, we apply our estimates of $q_{t,n}^b$ and $\sigma_{t,n}^2$ from the first stage. This leaves us with two types of economic parameters to estimate: an efficiency coefficient $\alpha_{i,n}$ for each bidder $i$ and auction $n$ and a common CARA coefficient $\gamma_n$ for each auction $n$. Variation in $\alpha_{i,n}$ reflects differences in bidder size, capacity at the time of bidding, and specialization for the particular project at hand. To reflect this, we project $\alpha_{i,n}$ onto a bidder fixed effect and a regression model of bidder-auction characteristics:

$$\alpha_{i,n} = \alpha_i + \beta_\alpha X_{i,n}.$$ Variation in $\gamma_n$ reflects the CARA approximation given different stakes involved with different auctions, as well as different compositions of participating bidders. To capture this, we project $\gamma_n$ onto a vector of auction characteristics, including auction-level averages of auction-bidder features among the bidders involved:

$$\gamma_n = \gamma_0 + \beta_\gamma X_n.$$  

Assumption 1 implies that for each item-bidder-auction tuple, the residual difference $\nu_{t,i,n}$ between the unit bid observed in our data and the optimal bid given by Equation (6) has a mean of zero, and is uncorrelated with either the identity or the characteristics of the bidder and the item being bid on. Given this exclusion restriction, we estimate the second stage parameters $\gamma_0, \beta_\gamma, \alpha_i$ and $\beta_\alpha$ through a system of moment conditions:

$$\mathbb{E}_n[\tilde{\nu}_{t,i,n} \cdot Z_{t,i,n} | X_{t,n}, X_{i,n}] = 0,$$

where $Z$ is each of the following instruments, and their interactions:$^{12}$

- Indicator for being a “top skewed item”
- Indicator for unique firm IDs
- The bidder-auction feature vectors that comprise $X_{i,n}$

Here, “top skewed items” are items that were flagged by the DOT’s Engineering Office as being prone to having especially high or low bids. The list of these items largely corresponds to the most frequent strongly over-/under- bid items in our data. While not completely

$^{12}$We write $\tilde{\nu}_{t,i,n}$ rather than $\nu_{t,i,n}$ because Eq. (6) invokes the optimal score $s_{i,n}^*$, which is not directly observed. Instead, we observe $s_{t,n}^d = \sum_t b_{t,i,n}^d q_{t,i,n}$, which is also noisy. As we explain in detail in Appendix D.1.2, this additional noise cancels out in expectation when we plug $s_{t,n}^d$ into Equation (6) when forming our moments, and so does not bias estimation.
symmetric, these items are typically observed with both over-bids and under-bids in different contexts. According to our model, the variation in these bids is reflective of the level of bidders’ responses to the uncertainty regarding the quantities of these items—both in absolute terms and relative to the remainder of the project. As such, “top skewed items” are particularly reflective of the balance of cost efficiency and risk aversion.

Each of our moments asymptotes in the number of auctions, capturing a slice of variation in the data. Each instrument is a feature characterizing a group of optimal bids and (by assumption) satisfying the exclusion restriction. Each moment is therefore identified by the assumption that the sample average of measurement errors asymptotes to zero as the number of auctions increases, independently of the variation in the pertaining instrument.

More specifically, in order to leverage the variation across auctions that each bidder participates in, we construct moments for the average measurement error among all bids, and among “top skewed item” bids, across all of the auctions that each unique bidder participated in. Similarly, we build moments that capture the variation across auctions and the states of the bidders participating in them. These moments average interactions between measurement errors with each column of the bidder-auction feature matrix $X_{i,n}$ among all bids, and among “top skewed item” bids, across all bidders and auctions collectively. We describe our full estimation procedure in detail in Appendix D.

**Extensions** As we noted in Section 5.2, our estimation procedure requires only minimal assumptions on the competitive conditions of the auction game. Our identification strategy uses variation in the determinants of unit bids conditional on observed scores. In contrast to the C/GPV-style approach, this is agnostic to the exogeneity of the set of participating bidders in each auction. As such, our estimation results are robust to alternative assumptions regarding bidders’ beliefs about their opponents’ types and strategies.

However, our approach does require functional form assumptions on the structure of bidder utility and bidder beliefs about item quantities. While in principle, our approach can accommodate a higher dimensional parameter space in the second stage—and perhaps even a semi-parametric model of bidding—we found this to be unfeasible for our setting given the number of auctions and the heterogeneity of item portfolios across them in our data. However, to demonstrate the flexibility of our approach we present two alternative specifications in the Appendix: a model with two types of CARA coefficients—one for frequent bidders and one for infrequent ones—and a model with CRRA risk aversion.
7 Estimation Results

Our structural estimation procedure consists of two parts. In the first stage, we estimate the distribution of the ex-post quantity of each item conditional on its item-auction characteristics using Hamiltonian Monte Carlo. We present parameter estimates for the regression coefficients on the predicted quantity term \( \hat{q}^b_{t,n} \), as well as the variance term, \( \hat{\sigma}_{t,n}^2 \) in Table 10 in the appendix. A histogram of the resulting variance terms themselves is plotted in Figure 5, below. Prior to estimation, all item quantities were scaled so as to be of comparable value between 0 and 10. As demonstrated in the histogram, the majority of variance terms are between 0 and 3, with a trailing number of higher values. In addition, we demonstrate the model fit of our first stage in Figure 6 and Table 12 in the appendix.

Figure 5: A histogram of standard deviation estimates for each item \( t \) in each project \( n \)

In the second stage, we estimate a project-specific CARA coefficient \( \gamma_n \), as well as a bidder-auction specific efficiency type \( \alpha_{i,n} \) for every bidder-auction pair in our data using the GMM estimator presented in Section 6. We summarize the results in Tables 3, 4, and 5.\(^{13}\) Bootstrapped standard errors and confidence intervals are presented in Table 11 in the appendix. The median coefficient of risk aversion \( \gamma \) in our data is estimated to be about 0.044 when dollar values are scaled by $1,000. An individual with this level of risk aversion would require a certain payment of $22 to accept a 50-50 lottery to either win or lose $1,000 with indifference, and $2,132 to accept a 50-50 lottery to win or lose $10,000. However, there is substantial variation across project types and projects more generally.

In Table 4, we present summary statistics of our estimates of bidder-auction efficiency types. We break down the results by project type to highlight the differences between different types of construction. An efficiency of 1 would suggest that the bidder faces costs

\(^{13}\)Summaries are computed after winsorizing by 1% in order to best exposit the distributions.
Table 3: Summary statistics of $\gamma_n$ estimates by project type.

<table>
<thead>
<tr>
<th>Project Type</th>
<th>Mean</th>
<th>St Dev</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>0.071</td>
<td>0.114</td>
<td>0.024</td>
<td>0.044</td>
<td>0.073</td>
</tr>
<tr>
<td>Bridge Reconstruction/Rehab</td>
<td>0.050</td>
<td>0.060</td>
<td>0.022</td>
<td>0.026</td>
<td>0.057</td>
</tr>
<tr>
<td>Bridge Replacement</td>
<td>0.081</td>
<td>0.152</td>
<td>0.023</td>
<td>0.037</td>
<td>0.078</td>
</tr>
<tr>
<td>Structures Maintenance</td>
<td>0.070</td>
<td>0.080</td>
<td>0.035</td>
<td>0.048</td>
<td>0.073</td>
</tr>
</tbody>
</table>

Table 4: Summary statistics of $\alpha_i^n$ estimates by project type.

<table>
<thead>
<tr>
<th>Project Type</th>
<th>Mean</th>
<th>St Dev</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>0.943</td>
<td>0.233</td>
<td>0.837</td>
<td>0.975</td>
<td>1.099</td>
</tr>
<tr>
<td>Bridge Reconstruction/Rehab</td>
<td>0.939</td>
<td>0.220</td>
<td>0.842</td>
<td>0.974</td>
<td>1.077</td>
</tr>
<tr>
<td>Bridge Replacement</td>
<td>0.970</td>
<td>0.212</td>
<td>0.856</td>
<td>0.993</td>
<td>1.114</td>
</tr>
<tr>
<td>Structures Maintenance</td>
<td>0.905</td>
<td>0.262</td>
<td>0.794</td>
<td>0.946</td>
<td>1.077</td>
</tr>
</tbody>
</table>

exactly at the rates represented by MassDOT’s estimates. Our results show that the median bidder overall has an efficiency type of 0.975, consistent with estimates of bidder costs by previous papers.\textsuperscript{14} There is heterogeneity across project types, however. We estimate that the median bidder in a bridge replacement project has an efficiency type of about 0.993, suggesting that her material costs are 0.07% lower than the DOT’s estimates. The median bidder in structures maintenance projects, however, has an efficiency type of about 0.946, suggesting that she obtains costs that are 5.4% lower than the DOT cost estimates.

In Table 5, we present the ex-post markups for each bidder given their efficiency type:

$$\text{Markup} = \frac{\sum_t q_{t,n}^a \cdot (b_{t,i,n} - \alpha_{i,n} c_{t,n})}{\sum_t q_{t,n}^a \cdot (\alpha_{i,n} c_{t,n})}.$$  

This is the bidder’s total ex-post profit from the project, normalized by her total cost. The overall median markup for a bidder in our data set, overall, is about 17%, whereas the median markup for a winning bidder is 7%. Rather than summarize markups by project type, we split projects by the number of participating bidders in each auction. Although there is substantial heterogeneity within each group, markups are generally decreasing with the amount of competition. However, the median markup in auctions with fewer than 9 bidders is between 28% and 38% across the board. This suggests that while competitive

\textsuperscript{14}See Bajari, Houghton, and Tadelis (2014) and Bhattacharya, Roberts, and Sweeting (2014), for example.
forces are generally present, underlying differences across projects and the contractors that bid to fulfill them play a major role in determining contractors’ ability to profit. Note that this markup measure does not account for extra work orders, as our model does not identify the cost of fulfilling them. While this may help explain why some of the markup estimates on the lower tail are negative, it does not imply that our parameter estimates are biased. Any additional payments that may have been anticipated at the time of bidding would have influenced the bidders’ choice of equilibrium score. However, because these payments were not bid upon, their presence would not change the solution to the bid portfolio optimization problem conditional on the observed equilibrium score.

<table>
<thead>
<tr>
<th>Type of Bidder</th>
<th>Bidder Markups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>All</td>
<td>40%</td>
</tr>
<tr>
<td>Winning Only</td>
<td>38%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Num Bidders in Auction</th>
<th>Mean</th>
<th>SD</th>
<th>25%</th>
<th>Median</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>81%</td>
<td>131%</td>
<td>16%</td>
<td>30%</td>
<td>77%</td>
</tr>
<tr>
<td>3</td>
<td>87%</td>
<td>205%</td>
<td>7%</td>
<td>28%</td>
<td>90%</td>
</tr>
<tr>
<td>4</td>
<td>77%</td>
<td>157%</td>
<td>2%</td>
<td>38%</td>
<td>95%</td>
</tr>
<tr>
<td>5</td>
<td>61%</td>
<td>103%</td>
<td>5%</td>
<td>35%</td>
<td>78%</td>
</tr>
<tr>
<td>6</td>
<td>60%</td>
<td>119%</td>
<td>9%</td>
<td>30%</td>
<td>67%</td>
</tr>
<tr>
<td>7</td>
<td>42%</td>
<td>63%</td>
<td>6%</td>
<td>30%</td>
<td>59%</td>
</tr>
<tr>
<td>8</td>
<td>43%</td>
<td>80%</td>
<td>7%</td>
<td>31%</td>
<td>56%</td>
</tr>
<tr>
<td>9</td>
<td>31%</td>
<td>74%</td>
<td>-8%</td>
<td>13%</td>
<td>40%</td>
</tr>
<tr>
<td>10+</td>
<td>10%</td>
<td>52%</td>
<td>-16%</td>
<td>-2%</td>
<td>18%</td>
</tr>
</tbody>
</table>

Table 5: Summary statistics of estimated bidders’ markups by number of participating bidders

We defer a demonstration of the goodness of fit of our structural model to the appendix. Figure 7 plots the unit bids predicted by our model on the x-axis, and the unit bids observed in our data on the y-axis. Figure 8a plots a quantile-quantile plot of our model-predicted bids against the data bids. While bid predictions are not perfect, the correspondence between predictions and data is quite good. Table 13 presents a regression analysis of the predictiveness of our model fit on the observed data. Our model fit predicts data bids with an R-squared of 0.865.
8 Counterfactuals

In Section 4, we argued that the bids observed in MassDOT bridge procurement auctions are suggestive of risk aversion. In Sections 5 and 6, we showed that under risk aversion, the unit bids alone could be used to identify parameters for bidder efficiency types and risk aversion, independently of strong assumptions about the competitive environment. In this section, we argue that the level of risk and risk aversion exhibited in these auctions is substantial, and consider the effectiveness of several counterfactual policies to lower DOT costs. Although we did not require assumptions on bidders’ beliefs about their opponents for estimation, we do need to take a stance on this for the purpose of counterfactuals. As the focus of our paper is on project risk and risk aversion, we assume that efficiency types are independent private values, drawn from a commonly known distribution for each auction. As we demonstrate in Figure 8b in the appendix, the equilibrium score predicted according to this model align closely with those observed in the data.

Constructing Counterfactual Equilibria In order for a vector of scores \( \{s_{i,n}\}_{i \in I(n)} \) to constitute an equilibrium for an auction \( n \), it must be that the expected utility for participating in the auction is maximized for each bidder \( i \) at \( s_{i,n} \). As discussed in Section 5, bidder \( i \)'s expected utility for a given score \( s \) is given by:

\[
EU(s, \alpha_{i,n}) = [1 - \exp(-\gamma_n \cdot CE(b_{i,n}(s), \alpha_{i,n}))] \times \prod_{j \neq i} [1 - H_{j,n}(s)],
\]

where the certainty equivalent function \( CE(b_{i,n}(s), \alpha_{i,n}) \) is given in Eq. (3) and the vector \( b_{i,n}(s) \) is given by the solution to the portfolio problem in (5). The term \( \prod_{j \neq i} [1 - H_{j,n}(s)] \) corresponds to the probability that \( s \) is the lowest score submitted under the distribution of opponent scores. We assume that bidder efficiency types are drawn IID from a common auction-specific distribution with CDF \( F_n \), that is calibrated by fitting the estimates from Section 7 to a truncated log-normal. As such, in the unique monotonic equilibrium, the probability of winning with \( s \) is equal to the probability that the inverse of \( s \) is the lowest efficiency type: \( (1 - F_n(s^{-1}(s)))^{I(n)-1} \). The function that maps efficiency types \( \alpha \) to equilibrium scores \( s_n(\alpha) \) in each auction is characterized by the first order condition of Eq. (9) with respect to \( s \). To evaluate each counterfactual, we solve the resulting differential equation in each auction \( n \), solve the portfolio problem for every \( (\alpha, s_n(\alpha)) \) pair to obtain the equilibrium bid vector \( b_{\alpha}(s_n(\alpha)) \) and integrate the resulting ex-post DOT cost function \( (b_{\alpha}(s_n(\alpha)) \cdot q^a) \) with respect to the first order statistic of the \( \alpha \) distribution. To evaluate bidder welfare, we integrate the bidder expected utility function rather than ex-post DOT costs. See Appendix A for more details.
Scaling Auctions as Insurance  While scaling auctions are widely used in many parts of public procurement, they are not ubiquitous. Even within MassDOT, there is heterogeneity: in 2007, the division responsible for public transportation switched from scaling auctions to a lump sum format in which contractors submit a single total bid for completing the project under auction. Lump sum auctions have some attractive properties. They may require less detailed specification plans from the DOT engineers, and they pass the incentive to minimize costs onto the contractor, thereby reducing the scope for moral hazard.

However, in the context of bridge auctions—where DOT officials are able to monitor work effectively enough to eliminate moral hazard concerns—scaling auctions provide an important mechanism for containing DOT costs. Lump sum auctions require bidders to pre-commit to a payment at the time of bidding, leaving them liable for all unforeseen changes. By contrast, scaling auctions compensate bidders for whatever item quantities are actually used, and they allow bidders to hedge their bets through portfolio optimization. Equation (10) compares the certainty equivalent that a bidder submitting the same bid vector $b$ would expect under a scaling auction, and under a lump sum auction.

\[
CE(b, \alpha) : \text{Scaling Auction} = \sum_t \left( b_t q_t^b - \alpha c_t q_t^b \right) - \frac{\gamma \sigma_t^2}{2} \left( b_t - \alpha c_t \right)^2
\]

\[
CE(b, \alpha) : \text{Lump Sum Auction} = \sum_t \left( b_t q_t^e - \alpha c_t q_t^b \right) - \frac{\gamma \sigma_t^2}{2} \left( \alpha c_t \right)^2
\]

(10)

Not only is the risk term for each item smaller under the scaling format, but the bidder is able to temper the contribution of items with high variance by bidding close to her cost. In this sense, scaling auctions provide insurance against project uncertainty that is unavailable in lump sum auctions. In equilibrium, this insurance makes winning each auction much more attractive, thereby incentivizing each contractor to bid more aggressively.

Our simulations show that the amount of insurance provisioned by MassDOT bridge auctions is substantial. Moving to a lump sum format would increase DOT payments to the winning bidder by nearly 68% in the median auction in our dataset. Given the scope of the projects in our data, this amounts to additional spending of over $950,000. Furthermore, the distribution of added costs is fat-tailed: while the 25th percentile of projects would cost 16% more under a lump sum format, the 75th percentile would cost 161% more.

Lump Sums with Renegotiation  The lump sum simulations above assume that—as is the case in scaling auctions—there is no ex-post renegotiation. As such, bidders are liable for the entirety of unforeseen project costs no matter how large they become. In practice, however, bidders may be able to recoup some of their costs by negotiating for additional payments based on ex-post quantity realizations. To account for this possibility, we consider a model in which bidders expect to be able to recoup a percentage $\lambda$ of earnings lost due
to unforeseen project changes—for instance through a Nash Bargaining negotiation. In order to credibly convey their lost value from the project, the bidders are forced to credibly show that their claimed ex-post project total could be generated by unit prices that are consistent with their initial lump-bid. As such, each bidder expects to be paid her score (as in the basic lump sum case), plus $\lambda$ of the ex-post quantity differential of each item multiplied by the item’s unit price. Bidders internalize the additional ex-post payments at the time of bidding, and so they choose unit prices to maximize their certainty equivalent:

$$CE(b, \alpha) : \text{Lump Auction w Renegotiation}$$

$$ \sum_i b_i q^e_i + (\lambda b_i(q^b_i - q^e_i)) - \alpha c_i q^b_i - \frac{\gamma \sigma^2}{2} (\lambda b_i - \alpha c_i)^2. \quad (11) $$

Comparing Equation (11) to Equation (10), it is clear that renegotiation significantly reduces the amount of risk that bidders are exposed to. This is not only because bidders are partially reimbursed for every item, but also because pre-committing to unit prices allows the bidders to optimize their balance of risks similarly to a scaling auction. In our simulations, we find that moving from a scaling auction format to a lump sum format with 2:1 renegotiation ($\lambda = 0.33$) would only increase DOT costs by 7% or $96,161 for the median auction. If the bidder is able to bargain with equal weight ($\lambda = 0.5$), the increase to median DOT costs is only 2.61% or $31,823. Moreover, while the 25th percentile auction would see increased costs of 8.83% ($136,484), the 75th percentile would save 1.61% ($13,993), suggesting that shifting a small amount of risk onto bidders might push them to reduce the riskiness of their portfolio choices in a beneficial way. We explore a similar idea through a minimum unit reservation price further in this section.

**The Cost of Uncertainty Under Scaling Auctions** Our lump sum results suggest not only that scaling auctions provide substantive insurance for the bidders in our data, but also that the amount of project uncertainty that bidders face is large. As such, a direct method to reduce DOT costs may be to simply lower ex-ante uncertainty—for instance, by improving inspection directives and engineer training. To test the potential for a policy of this sort to be effective, we consider an extreme counterfactual in which the DOT is able to perfectly predict exactly what quantity of each item will be required to fulfill each project. The ex-post correct quantities are then posted publicly from the beginning so that $q^e = q^a$. All bidders know that these quantities are correct and so they do not anticipate any further uncertainty: $q^b = q^a$ and $\sigma^2 = 0$.

Disclosing ex-post quantities to bidders at the start of an auction has two effects. First, it trivializes the portfolio problem: with no project uncertainty, there is no need to hedge or skew. On the other hand, it grants bidders access to the exact ex-post quantities—
rather than sophisticated estimates thereof—allowing the bidders to perfectly maximize their earnings from an ex-post perspective at the DOT’s expense. As we are primarily interested in quantifying the cost of uncertainty itself, we first compare the no-risk counterfactual against a baseline in which the bidders’ predictions align with the ex-post quantities \((q^b = q^a)\), but the level of uncertainty \(\sigma^2\) is unchanged. We find substantial reductions: DOT spending would decrease by about 13.5% or $162,969 in the median auction. The 75th percentile auction would save about $288,597 (20%)—fairly close the 75th percentile ex-post cost overrun metric from Table 1—while the 25th percentile would save $90,396 (7%). This suggests that when quantity realizations match predictions closely, a substantial portion of spending can be attributed to risk premiums, even net of competition and the benefits of scaling auctions.

However, a policy of simply reducing risk may not hold up to practical considerations. When we compare the no-risk counterfactual to the baseline with the estimated predictions \(q^b\) from Section 7, the median DOT saving is only 0.28% or $3,579. The 75th percentile of savings is a mere 1.8% or $21,205, and the 25th percentile is a loss of 0.9% or $13,468. This is not entirely surprising given the symmetry of cost over-runs and under-runs in Table 1. Our evaluation is with respect to ex-post realizations of quantities, whereas bidder optimization is with respect to ex-ante predictions. For items that have high variance, good ex-ante predictions will still be off half of the time, and the resulting bidder optimization will be sub-optimal from an ex-post perspective. Ex-post bidder gains are DOT losses and so when the no-risk counterfactual gives bidders the ability to perfectly optimize, the net effect on DOT costs is mitigated or even negative in some cases. In sum, while reducing uncertainty is still beneficial overall, it may not be worth the cost of training or additional bureaucracy.

**Item-Level Reservation Prices** Our no-risk counterfactual suggests that a substantial risk premium remains in MassDOT scaling auctions. However, this premium is an equilibrium outcome subject to auction rules. In January 2017, MassDOT attempted to require a minimum reservation price for every unit bid in a selection of projects due to bid skewing concerns. SPS New England, Inc.—a frequent bidder in our data—protested, arguing that such rules preclude the project from being awarded to the lowest responsible bidder. The Massachusetts Assistant Attorney General ruled in favor of the contractor on August 1, 2017.

In the context of our model, a minimum bid restriction of this sort is captured by a simple modification to the bidders’ portfolio problem in Equation (5): whereas in the baseline, optimal unit bids are constrained to be non-negative, a minimum bid restriction requires that the unit bid for each item be above a strictly positive reservation value. In this sense, SPS has a point: our model shows that bidders with higher \(\alpha\) types will generally place
higher unit bids on items that under-run. As such, a minimum bid constraint is less likely to bind for the highest \( \alpha \) types, leaving their equilibrium bids (and scores) closer to their unrestricted levels. Bidders with lower \( \alpha \) types, by contrast, are more likely to be bound by the constraint and submit relatively higher scores in equilibrium.\(^{15}\)

However, in practice this competitive effect is dominated by another feature of equilibrium bidding under a minimum bid restriction. Absent any restrictions, the bidders’ portfolio problem maximizes their expected utility from winning the auction, but it does not necessarily minimize the risk that they take upon themselves. Indeed, as in standard asset pricing, it is typically optimal to take some risk in pursuit of high returns. A minimum bid restriction changes this calculus by making some types of risk-taking impossible. The restriction only binds when there are items that bidders are willing to get minimal compensation for. Bidding close to zero on these items entails risk, as the bidder may be liable for a substantial uncompensated cost if the item is over-used. Forcing bidders to bid higher on these items therefore reduces risk, which adds value to the bidders’ expected utility. As such, although (holding the score fixed) expected utility decreases for bidders under a minimum bid constraint, it does not decrease dramatically. Consequently, the increase in equilibrium scores for bidders bound by the constraint is also subdued.

Furthermore, the restricted optimal bids are beneficial to the DOT. Whereas the unconstrained portfolio program allows bidders to optimize their expected utility by balancing profit maximization against risk minimization, the restricted problem forces bidders to collectively prioritize lowering risk. To see this, note that in order for there to be an item that is underbid, there must also be another item that is overbid using the additional “slack” in summing up to the bidder’s competitive score. Given the forces described above, equilibrium scores under the minimum bid restriction do not increase enough to offset the increase in unit bids on “low value” items. Consequently, the bidders are forced to also decrease their bids on the “high value” items to compensate. As a result, bidders submit portfolios that are aggregately less risky: both “low value” and “high value” items are bid closer to cost.

To evaluate a minimum bid restriction in our data, we consider a policy in which unit bids are required to be no smaller than 25% of the DOT estimated market rate for each item.\(^{16}\) Our simulations show that under this policy, the DOT would expect to save $28,042 or 2.28% in the median auction, with a 25th percentile of $12,895 (0.74%) and a 75th percentile of $55,141 (5.02%). Comparing a baseline with the minimum bid restriction to the

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\(^{15}\)If unit reservation prices are too high relative to the distribution of bidder types, it is possible that no symmetric equilibrium will exist. However, this will generally not occur for restrictions in the range considered by the DOT, and does not occur in our simulations.

\(^{16}\)The simulations discussed here use \( q^b = q^a \) for the sake of exposition. The savings under the estimated \( q^b \) are smaller but the qualitative pattern of reducing risk persists.
no-uncertainty ideal, we find a remaining risk premium of 10.63%—a 21 percent reduction—for the median auction. While this constitutes a modest reduction in expected payments to the winning bidder, the minimum bid restriction policy is cheap and simple to implement. As such, it may serve as a practical mechanism to further reduce costs in MassDOT scaling auctions.

9 Conclusion

This paper studies the bidding behavior of construction firms that participate in scaling procurement auctions hosted by the Massachusetts Department of Transportation. We develop a model of equilibrium bidding by risk averse bidders that are collectively better informed than the auctioneer. As noted previously in the literature, informed bidders are incentivized to strategically *skew* their bids, placing high bids on items they predict will over-run the DOT’s quantity estimates and low bids on items they predict will under-run. Risk averse bidders go further—by balancing their bid portfolio across items with different levels of uncertainty, they limit their exposure to the risk of unexpected changes in the quantities ultimately needed to complete a project.

We present evidence that bidding in our setting is consistent with these predictions: holding all else fixed, items that over-run MassDOT’s predictions have higher bids on average, while items that bear higher uncertainty have lower bids. Furthermore, we argue that accounting for risk aversion has significant implications for policy design. If the bidders were risk neutral, common policies such as switching to a lump sum format or investing in engineer training to reduce uncertainty would not change MassDOT spending in equilibrium. If the bidders are risk averse, however, these policies have theoretically ambiguous, potentially large consequences.

To assess the cost due to risk in our context, and evaluate the effectiveness of these different policies empirically, we estimate the parameters underlying our model. We then simulate the equilibrium bid vector at the item-bidder-auction level for every type of bidder in each of our auctions under the aforementioned counterfactual policies. We estimate that the level of uncertainty in our setting is large—it entails up to a 13.5% premium on payments to the winning bidder in the median auction. However, an effort to reduce uncertainty may not reduce costs very much in practice, as this would also improve bidders’ ability to maximize their ex-post revenue. Moreover, switching to a lump sum auction would be very costly—69% more for the median project—as this would expose bidders to full liability for unexpected changes in the project specification.

Viewed in this light, scaling auctions allow MassDOT to insure bidders against inevitable
shocks due to underlying conditions that are unearthed at the time of construction. It may be possible to improve this further, however—even with a simple, low-cost policy. We find that a 2017 proposal to enforce a minimum on the unit bids that bidders are allowed to submit could cut the risk premium by over a fifth. While our framework enables evaluating farther-reaching policies as well, such as an adaptation of the multi-stage optimal mechanism established by Maskin and Riley (1984) and Matthews (1987). We leave this for future work.
References


A Scaling Equilibrium Construction

Inputs: For each counterfactual we take the following auction-specific objects as inputs. We suppress the auction index \( n \) in notation for exposition.

- \( q^e \): DOT quantity estimates
- \( q^b \): Bidder quantity predictions
- \( \sigma^2 \): Bidder predictions’ variances
- \( c \): DOT cost estimates
- \( I \): The number of participating bidders
- \( \gamma \): Auction-wide risk aversion coefficient
- \( F(\alpha) \) and \( f(\alpha) \): Cdf and pdf of the distribution of efficiency types

The objects \( q^e, c \) and \( I \) are taken straight from data provided by the DOT. The remaining objects are estimates derived from Section 7. The estimates of \( q^b \) and \( \sigma^2 \) are taken directly from the first stage of our estimation procedure. For exposition of the role of risk itself in counterfactuals, we substitute the realized ex-post quantities \( q^a \) for bidder predictions.

We take the estimate for \( \gamma \) directly from the second stage of our estimation procedure. For the distribution of \( \alpha \) types, we fit the estimated values of \( \alpha_{i,n} \) to a truncated log-normal distribution with a mean that depends on project characteristics, and a project-type-specific variance:

\[
\alpha_n^i \sim \text{LogNormal}(\mu_n^\alpha, \sigma_n^\alpha^2)
\]

where \( \mu_n^\alpha = X_n \beta_\alpha \) and \( \sigma_n^\alpha \) is project-type specific.\(^{17}\) We estimate \( \beta_\alpha \) and \( \sigma_n^\alpha \) from the estimated distribution of \( \alpha \) types, using Hamiltonian Monte Carlo with MC Stan. We continue to use \( F(\alpha) \) and \( f(\alpha) \) to refer to the cdf/pdf of this distribution for the remainder of the derivation for notational convenience.

Equilibrium Construction: As discussed in Section 8, the expected utility that a bidder \( i \) receives for participating in an auction is given by:

\[
EU(s, \alpha_i) = \left[ 1 - \exp(-\gamma \cdot CE(b_i(s), \alpha_i)) \right] \times \prod_{j \neq i} \left[ 1 - H_j(s) \right],
\]

\(^{17}\)It is not possible to statistically infer both the mean/variance parameters and the bounds of the lognormal domain. As such, we used 0.8 times the lowest observed (estimated) type in each auction as that auction’s lower bound, and 1.2 times the highest observed type as the upper bound. For extreme cases, we truncated by 0.5 from below and 3.5 from above. These decisions do not have a qualitative bearing over our results, but do influence the precise counterfactual predictions.
where \( CE(b_i(s), \alpha_i) \) is given by
\[
CE(b_i(s), \alpha_i) = \sum_{t=1}^{T} q_t^h(b_{i,t}(s) - \alpha_i c_t) - \frac{\gamma \sigma_t^2}{2} (b_{i,t}(s) - \alpha_i c_t)^2,
\]
(14)
and the vector \( b_{i,n}(s) \) is given by the solution to the portfolio problem:
\[
b_i^*(s) = \max_{b_i} \left[ \gamma \sum_{t=1}^{T} q_t^h(b_{i,t} - \alpha_i c_t) - \frac{\gamma \sigma_t^2}{2} (b_{i,t} - \alpha_i c_t)^2 \right]
\]
s.t. \( \sum_{t=1}^{T} b_{i,t} q_t^e = s \) and \( b_{i,t} \geq 0 \) for all \( t \).

Note that as unit bids cannot be negative, the portfolio problem in Equation (15) does not have a closed form solution, and must be solved numerically. In every instance that optimal bids must be evaluated, we compute them through a custom algorithm detailed in Appendix C.

We assume that bidder efficiency types \( \alpha \) are drawn IID from the auction-wide distribution. As such, there is a unique symmetric equilibrium in monotonic strategies. Writing the equilibrium bidding function: \( \sigma : [\alpha, \overline{\alpha}] \rightarrow \mathbb{R} \), we can rewrite the probability that \( i \) will win the auction under bid \( s \) as follows:
\[
\prod_{j \neq i} [1 - H_j(s)] = \prod_{j \neq i} [Pr(s < \sigma_j(\alpha_j))]
\]
(16)
\[
= \prod_{j \neq i} [1 - F(\sigma_j^{-1}(s))]
\]
(17)
\[
= [1 - F(\sigma^{-1}(s))]^{n-1},
\]
(18)
where Equation (17) follows from the monotonicity of the equilibrium bidding function, and Equation (18) follows from symmetry, by which all bidders use the same equilibrium bidding function.

To simplify notation, we rewrite Equation (25) as follows:
\[
EU(\sigma(\alpha_i), \alpha_i) = V(\sigma(\alpha_i)) \times [1 - F(\sigma^{-1}(\sigma(\alpha_i)))]^{n-1}.
\]
(19)

This is a concave function, and so the optimal score at each \( \alpha_i \) is characterized by the first order condition:
\[
\frac{\partial EU(\sigma(\alpha_i), \alpha_i)}{\partial s} = 0
\]
(20)
where
\[
\frac{\partial EU(\sigma(\alpha_i), \alpha_i)}{\partial s} = \frac{\partial}{\partial s} V(\sigma(\alpha_i)) \times [1 - F(\sigma^{-1}(\sigma(\alpha_i)))]^{n-1} +
V(\sigma(\alpha_i)) \times \frac{\partial}{\partial s} [1 - F(\sigma^{-1}(\sigma(\alpha_i)))]^{n-1}.
\]
Writing $CE(\sigma(\alpha))$ as shorthand for $CE(b(\sigma(\alpha)), \alpha)$, the derivative of $V(\sigma_i(\alpha))$ is as follows:

$$V(\sigma(\alpha)) = 1 - \exp[-\gamma CE(\sigma(\alpha))]$$

$$\frac{\partial}{\partial s} V(\sigma(\alpha)) = \gamma \frac{\partial}{\partial s} CE(\sigma(\alpha)) \times \exp[-\gamma CE(\sigma(\alpha))],$$

where

$$\frac{\partial}{\partial s} CE(\sigma(\alpha)) = T \sum_{t=1}^{T} \left[ \frac{\partial b^*_t(\sigma(\alpha))}{\partial s} \left( q^b_t - \gamma \sigma^2_t (b^*_t(\sigma(\alpha)) - \alpha c_t) \right) \right].$$

Here the derivative of $b_t(\sigma(\alpha))$ is taken with respect to the solution of the portfolio problem in Equation (15). We compute it through a forward-mode auto-differentiation function directly from our algorithm.

The derivative of the second part of Equation (19) is given through the product rule and:

$$\frac{\partial}{\partial s} [1 - F(\sigma^{-1}(\sigma(\alpha))))] = [-f(\sigma^{-1}(\sigma(\alpha))))] \times \frac{1}{\sigma'(\sigma^{-1}(\sigma(\alpha))}$$

$$= -f(\alpha) \sigma'(\alpha).$$

(22)

To find the equilibrium bidding function, we solve the differential equation in Equation (20) using stiff ODE methods implemented by Rackauckas and Nie (2017). The ODE is defined with respect to an initial boundary condition in which the worst (highest) $\alpha$ type receives zero utility upon winning. We find the score that generates this condition when the worst type (like all others) uses our portfolio maximization program to choose his bids at any score.

**Lump Sum Equilibria**  As noted in Section 8, the lump sum problem is identical to the scaling auction problem except for the formulation of the certainty equivalent. Here there is no portfolio problem and the certainty equivalent is given directly by the choice of the bidder’s score:

$$CE_{\text{Lump}}(s, \alpha_i) = s - \left[ \sum_{t=1}^{T} q^b_t \alpha_i c_t + \frac{\gamma \sigma^2_t}{2} (\alpha_i c_t)^2 \right]$$

(23)

With $\lambda$-renegotiation:

$$CE_{\lambda,\text{Lump}}(s, \alpha_i) = \sum_{t=1}^{T} \left[ (\lambda q^b_t + (1-\lambda)q^c_t) \cdot b_{i,t}(s) - q^b_t \cdot (\alpha_i c_t) \right] - \frac{\gamma \sigma^2_t}{2} (\lambda b_{i,t}(s) - \alpha_i c_t)^2.$$

(24)

**Minimum Bid Restrictions**  The minimum bid restriction operates simply by changing the constraint $b_t \geq 0$ to $b_t \geq 0.25 \times c_t$ in Equation (15). The resulting program can be solved directly using the algorithm in Appendix C.
B Robustness to EWOs and Moral Hazard

B.1 Extra Work Orders

The counterfactual analysis in Section 8 assumes that bidders do not anticipate any profits from extra work orders (EWOs). However, as we show in Table 1, EWOs add about 10% to a typical project’s total cost. As we do not observe quantities or bids for EWO, we cannot identify the distribution of profits that bidders may anticipate. However, we can simulate the effect of EWOs on bids in our data under some additional assumptions. For simplicity, we assume that bidders in a given project anticipate a fraction $\mu$ of the total EWO value $R$ observed for that project in our data. We consider $\mu = 10\%$ and $\mu = 50\%$.

To construct each equilibrium, we extend the derivation in Appendix A as follows. The expected utility that a bidder $i$ receives for participating in an auction is given by:

$$EU(s, \alpha_i) = [1 - \exp(-\gamma \cdot [\mu \cdot R + CE(b_i(s), \alpha_i)])] \times \prod_{j \neq i} [1 - H_j(s)],$$

where as before, $CE(b_i(s), \alpha_i)$ is given by

$$CE(b_i(s), \alpha_i) = \sum_{t=1}^{T} q_t^h(b_{i,t}(s) - \alpha_i c_t) - \frac{\gamma \sigma_t^2}{2} (b_{i,t}(s) - \alpha_i c_t)^2,$$

and the vector $b_{i,n}(s)$ is given by the solution to the portfolio problem:

$$b_{i}^*(s) = \max_{b_i} \left[ \gamma \sum_{t=1}^{T} q_t^h(b_{i,t} - \alpha_i c_t) - \frac{\gamma \sigma_t^2}{2} (b_{i,t} - \alpha_i c_t)^2 \right]$$

s.t. $\sum_{t=1}^{T} b_{i,t} q_t^e = s$ and $b_{i,t} \geq 0$ for all $t$.

Note that nothing in the portfolio problem in Equation (27) is directly affected by $R$. However, the choice of equilibrium score will be different, as the value function in Equation (19) includes the addition of $\mu \cdot R$. Additional revenue from the EWO induces lower equilibrium scores as a cheaper portfolio can result in the same certainty equivalent. As such, equilibrium costs are lower once EWOs are taken into account. This results in smaller effects for our main counterfactuals: a 10% profit on EWO earnings corresponds to a median risk premium of 13.5%, and a median lump sum cost increase of 66.5%—a .1 and 1.9 percentage point decrease from the baseline, respectively. While 10% is closer to mark-ups observed on portfolios in our data, we also consider $\mu = 50\%$ as an upper bound on anticipated EWO values. Here, we find a slight increase in risk premiums, and a more substantial decrease in lump sum costs: only 55.1% for the median auction. While this is a considerable drop from our baseline, it does not greatly change our qualitative interpretation of the results.
B.2 Moral Hazard

Although lump sum auctions induce more risk on bidders, they also provide an incentive for the winning bidder to minimize her costs. As MassDOT bridge procurement is heavily monitored, our baseline model treats ex-post bidder costs as an exogenous function of the quantity distributions involved in each project. However, if bidders were able to influence the ex-post realization of item quantities, the winning bidder would be able to make additional profit by over-using items she had bid high on. Knowing this, she might bid higher on items that will be easier to over-use.

While we cannot identify the presence of such moral hazard from our data, our framework can easily be extended to account for it in counterfactual simulations. Rather than just choose unit bids that sum to a particular score, bidders solve a joint portfolio optimization problem in which they first choose which item quantities to augment (and by how much), and then choose optimal unit bids. The extent of bidders’ quantity adjustments therefore depend on the bidders’ limitations in over-using different items—as well the inherent expected item quantities and uncertainties, and the bidders’ private efficiency types. Together these factors characterize a modified version of Equation (5). We consider the following three models that capture limitations on item over-use:

(a) **Quadratic Costs:** Bidders incur a quadratic cost of increasing each item quantity by a fraction $y_{i,t}$, scaled by an item-specific factor $\mu_t$:

$$
\bm_i^*(s) = \max_{b_i} \left[ \gamma \sum_{t=1}^{T} (1 + y_{i,t}) \cdot q_{i,t}^b \cdot (b_{i,t} - \alpha_i \cdot c_t) - \left[ \mu_t \cdot y_{i,t}^2 \right] - \frac{\gamma \sigma_t^2}{2} (b_{i,t} - \alpha_i \cdot c_t)^2 \right] \tag{28}
$$

s.t. $\sum_{t=1}^{T} b_{i,t} \cdot q_{i,t} = s$ and $y_{i,t}, b_{i,t} \geq 0$ for all $t$.

(b) **Budgeted Increments:** Bidders have a fixed budget $\bar{y}$ of percentage increases that they can spread across items. Items have different budget weights $\mu_t$:

$$
\bm_i^*(s) = \max_{b_i} \left[ \gamma \sum_{t=1}^{T} (1 + y_{i,t}) \cdot q_{i,t}^b \cdot (b_{i,t} - \alpha_i \cdot c_t) - \frac{\gamma \sigma_t^2}{2} (b_{i,t} - \alpha_i \cdot c_t)^2 \right] \tag{29}
$$

s.t. $\sum_{t=1}^{T} b_{i,t} \cdot q_{i,t} = s$ and $\sum_{t=1}^{T} y_{i,t} \cdot \mu_t = \bar{y}$ and $y_{i,t}, b_{i,t} \geq 0$ for all $t$.

(c) **Bulk Increments:** Bidders are able to choose $K \leq T$ items, each of whose quantity
they are able to increase by a fixed percentage \( \bar{y} \).

\[
b_i^*(s) = \max_{b_i} \left[ \gamma \sum_{t=1}^{T} (1 + \bar{y} \cdot z_{i,t}) \cdot q_i^b \cdot (b_{i,t} - \alpha_i c_t) - \frac{\gamma \sigma_t^2}{2} (b_{i,t} - \alpha_i c_t)^2 \right] \tag{30}
\]

\[
s.t. \quad \sum_{t=1}^{T} z_{i,t} = K \quad \text{where} \quad z_{i,t} \in \{0, 1\}, \tag{31}
\]

\[
\text{and} \quad \sum_{t=1}^{T} b_{i,t} \cdot q_i^c = s, \quad \text{with} \quad b_{i,t} \geq 0 \text{ for all } t.
\]

As each of these variants would require a separate implementation, we focus on model (c) with \( K = 1 \) as a gauge for the extent to which modeling moral hazard may shift our results. We chose model (c) rather than (a) and (b) for the following reasons. First, while the quadratic cost model (a) resembles the standard principal-agent moral hazard framework, its solution is highly dependent on the choices of \( \{\mu_t\} \). As we cannot identify \( \{\mu_t\} \) from our data, interpreting this model is less straightforward than the alternatives. Second, while the formulation of model (b) allows for continuous allocations of quantity increments \( y_{i,t} \) across different items, the optimal solution will typically allocate all of the budget \( \bar{y} \) to one item.\(^{18}\)

Thus our simulation of model (c) is equivalent to a variant of (b) with \( \mu_t = 1 \) for all \( t \).

We replicate our main counterfactual results under two bulk quantity increments: \( \bar{y} = 10\% \) and \( \bar{y} = 50\% \). In both cases, the risk premium (which in this case also includes the moral hazard premium, as absent risk there is no possibility for bidders to profitably increase quantities) is higher. The median risk premium increases from 13.6\% in our baseline to 16.4\% when bidders are able to over-use one item by 10\%, and to 26.7\% when bidders are able to over-use an item by 50\%. As typical over-runs in our data are on the order of 10\%, this suggests that if moral hazard had been present in our data, our estimates of the risk premium would likely be overestimated by several percentage points. Correspondingly, the cost of moving to a lump sum auction—in which there is no incentive to over-use items—decreases in the presence of moral hazard. The cost of switching a median auction to the lump sum format decreases from 68.4\% to 61.5\% with \( \bar{y} = 10\% \) and to 51.4\% with \( \bar{y} = 50\% \). This suggests that under a moderate amount of moral hazard, our main findings would underestimate the risk premium and overestimate the value of insurance by several percentage points. If extensive moral hazard were possible, our estimates would be farther off—by about 15 percentage points—but our general conclusions would hold.

\(^{18}\)A heuristic argument for this is as follows: quantity increases \( y_{i,t} \) only influence the objective function through increases to the linear term. As such, the optimal solution will choose the most valuable item in terms of expected profit and increase its quantity. As the score (budget) constraint may bind, it is possible that 2 items would be chosen, but this would only occur in edge cases.
C Model Extensions

C.1 Constant Relative Risk Aversion

Our baseline model assumed that bidders are endowed with a CARA utility function. While CARA is commonly used as a local approximation of general risk aversion—and does reasonably well at matching data in our simulations—it is likely to mischaracterize preferences when the stakes are an order of magnitude or more higher, as in our lump sum counterfactual. In this section, we discuss how our baseline model may be extended to a model of CRRA utility.

Primitives Suppose that bidders are instead risk averse with a standard CRRA utility function over their earnings from the project and a common constant coefficient of relative risk aversion $\gamma$:

$$u(\pi) = \frac{\pi^{1-\gamma}}{1-\gamma}. \quad (32)$$

As in our baseline, we will assume that all bidders in the same auction have the same level of (CRRA) risk aversion $\gamma$, but that the bidders are differentiated along their cost efficiency type $\alpha$. Similarly, bidder expectations for the quantities with which different items will be needed are noisy to different degrees.

Given the functional form of the CRRA model, a model of quantity uncertainty with Gaussian noise would be intractable. However, as is common in the asset pricing literature, we can make use of several identities and approximation techniques. The first such tool is the following fact, which shows that the expected utility of a lognormal random variable has a closed form. Suppose that $\log(x) \sim \text{Normal}(\mu, \sigma^2)$. Then:

$$E[u(x)] = \exp \left[ \frac{(1-\gamma)\mu + \frac{1}{2}(1-\gamma)^2\sigma^2}{1-\gamma} \right]. \quad (33)$$

In order to use this identity, we employ several assumptions. First, we assume that the ratio of ex-post to (DOT-predicted) ex-ante quantities is distributed lognormally:

$$\frac{q^x_i}{q^e_i} \sim \text{LogNormal}(\bar{p}_t, \sigma^2_t).$$

Modeling Expected Utility Considering a bidder with efficiency type $\alpha$, we write the bidder’s ex-post profit function:

$$\pi(b) = \sum_t q^a_t \cdot (b_t - \alpha c_t) = \sum_t q^c_t \cdot \left( \frac{q^a_t}{q^c_t} \right) \cdot (b_t - \alpha c_t). \quad (34)$$
Normalizing by \((s - \alpha \sum_k q_k^e c_k)\)—which we denote by \(R(s)\)—and reparametrizing, we can further write this:

\[
\pi(b) = \left( s - \alpha \sum_k q_k^e c_k \right) \cdot \left[ \sum_t \left( \frac{q_t}{q_t^e} \cdot (b_t - \alpha c_t) \right) \right] \tag{35}
\]

\[
= R(s) \cdot \left[ \sum_t \left( \frac{q_t}{q_t^e} \cdot (b_t - \alpha c_t) \right) \right] \tag{36}
\]

\[
= R(s) \cdot \left[ \sum_t w_t \exp(p_t) \right], \tag{37}
\]

where we use \(w_t \equiv \frac{q_t}{q_t^e} \cdot \frac{(b_t - \alpha c_t)}{R(s)}\) and \(p_t \equiv \log \left( \frac{q_t}{w_t} \right)\) in the latter equality.

While Equation (37) looks like a portfolio, it is not a lognormal random variable. However, taking logs and a second order Taylor approximation thereof, we achieve:

\[
\log(\pi(b)) \approx \log(R(s)) + \log \left( \sum_t w_t \right) + \sum_t \frac{w_t p_t}{\sum_k w_k} + \frac{1}{2} \sum_t \frac{w_t q_t^2}{\sum_k w_k} - \frac{1}{2} \sum_t \sum_r \frac{w_t w_r p_t p_r}{(\sum_k w_k)^2}
\]

To account for higher order terms, we apply a final technique from asset pricing. Using a continuous time approximation, we arrive at:

\[
\log(\pi(b)) \approx \text{Normal} \left( \log(R(s)) + \log \left( \sum_t w_t \right) + \frac{1}{2} \sum_t \frac{w_t q_t^2}{\sum_k w_k} - \frac{1}{2} \sum_t \sum_r \frac{w_t w_r}{(\sum_k w_k)^2} \cdot w^t \Sigma w, \right)
\]

where \(\Sigma\) is a diagonal matrix of variances, \(\sigma^2\) and \(w_t = q_t^e \cdot (b_t - \alpha c_t).\) Given this lognormal approximation of \(\pi(b),\) we can now compute expected utility. Plugging in the relevant terms and simplifying, we find:

\[
\log(E[u(\pi(b))]) \approx \log \left( \frac{[R(s)]^{1-\gamma}}{1-\gamma} \right) + \frac{1 - \gamma}{[R(s)]^2} \cdot \left[ \sum_t R(s) \cdot q_t^e \cdot (b_t - \alpha c_t) \cdot \left( \frac{\bar{p}_t}{\sigma_t^2} + \frac{1}{2} \sigma_t^2 + \frac{1}{2} \sum_{k:b_k>0} \frac{\bar{p}_k}{\sigma_k^2} \right) - \frac{\gamma}{2} \sum_t \left( q_t^e \right)^2 \sigma_t^2 \cdot (b_t - \alpha c_t)^2 \right]. \tag{38}
\]

### Computing Optimal Bids

As in the CARA case, the expected utility of profits upon winning in Equation (38) yields a portfolio problem. For a given score \(s,\) the bidder will choose a vector of bids \(b_t(s)\) that maximize Equation (38) subject to the condition that they are all non-negative and sum to \(s.\) Given the non-negativity constraints, there is again no closed form solution for optimal unit bids. However, at a given solution, every non-zero optimal unit bid has the following form:

\[
b_t^* = \alpha \cdot \left[ c_t - \frac{\sum_{k:b_k>0} q_k^e c_k}{q_t^e \cdot \sum_{k:b_k>0} \frac{1}{\sigma_k^2}} \right] + \frac{R(s)}{\gamma} \cdot \left[ \frac{\bar{p}_t}{\sigma_t^2} + \frac{1}{2} \sum_{k:b_k>0} \frac{\bar{p}_k}{\sigma_k^2} + \frac{1}{2} \right] + \left[ \frac{\sum_{k:b_k>0} \frac{1}{\sigma_k^2}}{q_t^e \cdot \sum_{k:b_k>0} \frac{1}{\sigma_k^2}} \right]. \tag{39}
\]
Estimation  Our estimation procedure mirrors that of the CARA model, making adjustments as necessary for the alternative set of assumptions. First, we estimate a first stage model of log-quantity-ratio predictions and variances. Denoting an auction by \( n \), we assume that the posterior distribution of each \( \log(q_{t,n}^a/q_{t,n}^e) \) is given by a statistical model that conditions on item characteristics (e.g. the item’s type classification), observable project characteristics (e.g. the project’s location, project manager, designer, etc.), and the history of DOT projects. In particular, we model the realization of the actual quantity of item \( t \) in auction \( n \) as:

\[
\log(q_{t,n}^a/q_{t,n}^e) = \hat{p}_{t,n} + \eta_{t,n}, \quad \text{where} \quad \eta_{t,n} \sim \mathcal{N}(0, \hat{\sigma}_{t,n}^2) \tag{40}
\]

such that

\[
\hat{p}_{t,n} = \beta_p X_{t,n} \quad \text{and} \quad \hat{\sigma}_{t,n} = \exp(\beta_\sigma X_{t,n}). \tag{41}
\]

As in the CARA case, we estimate this model with Hamiltonian Monte Carlo and use the estimates \( \hat{p}_{t,n} \) and \( \hat{\sigma}_{t,n} \) directly in the second stage of our procedure. Plugging the first stage estimates into Equation (39), we obtain a prediction of each bidder \( i \)'s unit bid for each item \( t \) given her observed score \( s_{i,n} \) in auction \( n \). We therefore adapt Assumption 1 once more and use the analogous moment conditions to the CARA case to estimate a project-wide CRRA parameter \( \gamma_n \) for every auction and an efficiency coefficient for every bidder-auction pair \( \alpha_{i,n} \). Given the structure of CRRA, bidder wealth is a relevant part of the utility function, that influences the optimal portfolio choice. While this again makes use of several approximations, we account for wealth by augmenting \( R(s) \equiv W + s - \alpha \sum_k q_k^c c_k \) throughout our model. We allow wealth \( W \) to vary from project to project, and estimate it by projecting onto the matrix of average project-bidder characteristics \( X_n \). We present our estimates in Table 6 below. Notably, our estimates of \( \alpha \) are higher than in the CARA case, so that implied markups are negative in many cases. However, the CRRA coefficients are mostly between 0.6 and 0.8, which is within the region found in related studies. See pg 133 of Campo, Guerre, Perrigne, and Vuong (2011) for a discussion.

<table>
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<tr>
<th>Parameter</th>
<th>Mean</th>
<th>SD</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
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<tr>
<td>( \alpha )</td>
<td>1.215</td>
<td>0.163</td>
<td>1.088</td>
<td>1.185</td>
<td>1.353</td>
</tr>
<tr>
<td>( \gamma )</td>
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<td>0.101</td>
<td>0.638</td>
<td>0.692</td>
<td>0.756</td>
</tr>
<tr>
<td>( W )</td>
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<td>135.568</td>
<td>140.988</td>
<td>235.571</td>
<td>333.681</td>
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</table>

Table 6: CRRA Estimate Summary Statistics Across All Projects
Table 7: Summary of Counterfactual Changes in Expected DOT Spending

<table>
<thead>
<tr>
<th>CF Type</th>
<th>∆ Cost Units</th>
<th>Mean</th>
<th>SD</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lump Sum %</td>
<td>-45.5</td>
<td>39.2</td>
<td>-50.2</td>
<td>-34</td>
<td>-22.2</td>
<td></td>
</tr>
<tr>
<td>Lump Sum $</td>
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<td>741,921</td>
<td>-1,149,156</td>
<td>-661,338</td>
<td>-341,346</td>
<td></td>
</tr>
<tr>
<td>No Risk (Correct q) %</td>
<td>-19.8</td>
<td>18.1</td>
<td>-26.2</td>
<td>-14.4</td>
<td>-7.6</td>
<td></td>
</tr>
<tr>
<td>No Risk (Correct q) $</td>
<td>-307,528</td>
<td>248,509</td>
<td>-421,562</td>
<td>-242,390</td>
<td>-147,097</td>
<td></td>
</tr>
<tr>
<td>No Risk (Estimated q) %</td>
<td>-41.7†</td>
<td>61.2†</td>
<td>-45.7</td>
<td>-24.4</td>
<td>-14.9</td>
<td></td>
</tr>
<tr>
<td>No Risk (Estimated q) $</td>
<td>-465,041†</td>
<td>315,305†</td>
<td>-595,512</td>
<td>-368,224</td>
<td>-234,416</td>
<td></td>
</tr>
</tbody>
</table>

Note: † refers to samples truncated by 5% to exclude extreme values

Computing Equilibrium Outcomes

We construct equilibria analogously to the CARA case. Here the analog of Equation (19) is given by:

\[
\text{EU}(\sigma(\alpha_i), \alpha_i) = \exp [(1 - \gamma) \cdot V(\sigma(\alpha_i), \alpha_i)] \cdot (1 - F(\sigma^{-1}(\sigma(\alpha_i)))) \\
+ W^{1-\gamma} \cdot F(\sigma^{-1}(\sigma(\alpha_i))),
\]

where

\[
V(\sigma(\alpha_i), \alpha_i) = \log (R(\sigma(\alpha_i), \alpha_i))+
\]

\[
\frac{1}{R(\sigma(\alpha_i), \alpha_i)} \cdot \left[ \sum_t q_t^e \cdot (b_t(\sigma(\alpha_i)) - \alpha_i c_t) \cdot \left( \bar{p}_t + \frac{1}{2} \sigma_t^2 \right) \right]
\]

\[
- \frac{\gamma/2}{R(\sigma(\alpha_i), \alpha_i)^2} \cdot \left[ \sum_t (q_t^e)^2 \sigma_t^2 \cdot (b_t(\sigma(\alpha_i)) - \alpha_i c_t)^2 \right],
\]

and \(b_t(\sigma(\alpha_i))\) is chosen according to the portfolio problem, with \(R = W + \sigma(\alpha_i) - \alpha_i \sum_r q_r^e c_r\).

The equilibrium function \(\sigma(\alpha)\) is thus given by the solution to the differential equation:

\[
\sigma'(\alpha) = \frac{f(\alpha)}{1 - F(\alpha)} \cdot \frac{1}{(1 - \gamma) \cdot \frac{\partial V(\sigma(\alpha), \alpha)}{\partial s}} \cdot \left[ 1 - \frac{W^{1-\gamma}}{\exp [(1 - \gamma) \cdot V(\sigma(\alpha), \alpha)]} \right]
\]

subject to the boundary condition that the highest \(\alpha\) type is indifferent between participating in the auction or not: \(V(\sigma(\bar{\alpha}), \bar{\alpha}) = \log(W)\). We present summary results for the lump sum and no-risk counterfactuals in Table 7.

C.2 Asymmetric Types

Our baseline model assumes that bidders are entirely homogeneous except for their private efficiency type \(\alpha\). While this greatly simplifies the analysis of counterfactual policies, our estimation strategy is able to accommodate models with multidimensional bidders fairly
easy. In this section, we discuss an extension to a model in which bidding firms of different sizes may have different CARA coefficients in each auction.

**Modeling and Estimation** While the focus of our paper is on the portfolio risk that bidders face within an auction, their exposure to that risk may depend on how large a role a given auction plays in their total profits. That is, firms that participate in many auctions and have larger portfolios may be less sensitive to the risk in a given project than firms that rely on a small number of projects. To account for this, we split our 25 unique bidders by the number of auctions that they participated in throughout our data. This splits the bidders roughly evenly: 14 firms are labeled as “frequent” bidders because they participated in 50 or more auctions, whereas the rest are “rare”. However, the distributions of participation are quite different: the average “frequent” firm participated in 123 auctions, whereas the average “rare” firm participated in 20.

As each bidder’s choice of optimal unit bids conditional on her equilibrium score does not depend on the types of any other bidders, Equation (6) and its empirical analogue remain unchanged if bidders have different values of $\gamma$ within an auction. Thus, for the purpose of estimation, we need only modify the empirical model for $\gamma$ in Section 6 to allow for variation by bidder size type:

$$
\gamma_{i,n} = \gamma_{g(i),n} + \beta_{g(i),n} X_{n},
$$

where $g(i) \in \{f, r\}$ denotes whether the bidder is a frequent bidder.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bidder Type</th>
<th>Mean</th>
<th>SD</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Frequent</td>
<td>0.946</td>
<td>0.231</td>
<td>0.846</td>
<td>0.978</td>
<td>1.092</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Rare</td>
<td>0.944</td>
<td>0.235</td>
<td>0.777</td>
<td>0.976</td>
<td>1.127</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Frequent</td>
<td>0.053</td>
<td>0.043</td>
<td>0.024</td>
<td>0.043</td>
<td>0.066</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Rare</td>
<td>0.074</td>
<td>0.092</td>
<td>0.025</td>
<td>0.051</td>
<td>0.086</td>
</tr>
</tbody>
</table>

Table 8: 2-Type Estimate Summary Statistics Across All Projects

Our results, summarized in Table 8, show that frequent bidders are not generally more cost efficient than rare bidders. While the median cost multiplier is about the same in both groups, the 25th percentile of rare firms is about 8% more efficient than the corresponding percentile of frequent firms, but the 75th percentile of rare firms is 3% less efficient than frequent firms. On the other hand, we find that frequent bidders exhibit slightly less risk aversion than rarer bidders. Neither of these results are surprising: frequent bidders likely specialize in work for the DOT. As such, they participate in auctions where they do not
have a particularly strong efficiency advantage. But their risks are also spread across more auctions, and so the weight of uncertainty in any particular project is lower.

**Computing Equilibrium Outcomes** While the estimation procedure is largely unchanged, computing equilibria when types are multidimensional is more demanding. We assume that bidder efficiency types $\alpha$ are drawn IID from the auction-wide distribution, and that bidder-type-specific CARA coefficients are publicly known. As such, there is an asymmetric equilibrium in monotonic strategies such that each bidder type bids according to the monotone function $\sigma_g : [\underline{\alpha}, \overline{\alpha}] \rightarrow \mathbb{R}$. Under a set of candidate strategies, the probability that $i$ will win the auction under bid $s$ is given by:

$$\prod_{j \neq i} [1 - H_j(s)] = \prod_{j \neq i} [Pr(s < \sigma_j(\alpha_j))]$$

$$= \prod_{j \neq i} [1 - F(\sigma_j^{-1}(s))]$$

$$= \prod_{j \neq i} [1 - F(\varphi_j(s))]$$

where the final equality relabels the inverse bid function for convenience. Applying logic analogous to our baseline specification to this more general framework, we derive the following set of differential equations:

$$\frac{\partial \varphi_f(\tilde{s})}{\partial s} = \frac{1 - F(\varphi_f(\tilde{s}))}{f(\varphi_f(\tilde{s}))} \cdot \left[ \frac{1}{(N_f + N_r - 1)} \left( N_r \cdot \frac{\partial^2 V(\varphi_r(\tilde{s}))}{\partial s^2} - (N_r - 1) \cdot \frac{\partial V(\varphi_f(\tilde{s}))}{\partial s} \right) \right]$$

$$\frac{\partial \varphi_r(\tilde{s})}{\partial s} = \frac{1 - F(\varphi_r(\tilde{s}))}{f(\varphi_r(\tilde{s}))} \cdot \left[ \frac{1}{(N_f + N_r - 1)} \left( N_f \cdot \frac{\partial^2 V(\varphi_r(\tilde{s}))}{\partial s^2} - (N_f - 1) \cdot \frac{\partial V(\varphi_f(\tilde{s}))}{\partial s} \right) \right]$$

where $V(\varphi_g(\tilde{s})) = 1 - \exp(-\gamma_g CE(\tilde{s}, \gamma_g, \varphi_g(\tilde{s})))$ (49)

and $CE(\tilde{s}, \gamma_g, \varphi_g(\tilde{s})) = \sum_{t=1}^{T} q_t b_t(\tilde{s}) - \varphi_g(\tilde{s}) c_t - \frac{\gamma_g \alpha_t^2}{2} (b_t(\tilde{s}) - \varphi_g(\tilde{s}) c_t)^2$. (50)

Here, the optimal bids $b_t^*(\tilde{s})$ are derived from the solution to the bidder’s portfolio problem in Equation (5), as in the baseline model. This set of differential equations characterizes an equilibrium subject to several boundary conditions. The particular boundary conditions that apply to a given auction depend on the auction format, the number of bidders of each frequency type that participate, and the magnitude of the difference between the bidders’ CARA coefficients.

---

19The existence of an equilibrium follows from Reny and Zamir (2004). Uniqueness may not be guaranteed and so our construction procedure may be thought of as imposing an equilibrium selection criterion.
There are three possible cases. The “standard” case requires that the highest and lowest $\alpha$ types of both frequency groups submit the same score, and that the highest $\alpha$ type of the more risk averse group (who has no chance of winning) earn a certainty equivalent of zero. This applies when there is only one bidder of the less risk averse group, or if project portfolio risk is irrelevant as in the no-risk counterfactual. However, as noted in Hubbard and Kirkegaard (2019), the standard conditions do not generally induce an equilibrium in an asymmetric auction with more than 2 bidders when the bidders have different supports for their value distributions. In our case, while both frequency groups have the same support of cost efficiency types $\alpha$, their different CARA coefficients induce different (overlapping) supports on the certainty equivalents of their portfolios at each score.

As such, when there are two or more bidders of each frequency group, different boundary conditions at either end apply. Suppose, for example that $\gamma_f < \gamma_r$, and let $\bar{\gamma}$ be the score that generates a zero certainty equivalent for the least competitive (highest $\alpha$) frequent bidder. No bidder submitting a higher score has a chance of winning, and so in equilibrium, any rare bidder submitting $\bar{\gamma}$ or higher earns a zero certainty equivalent as well. Thus, there is some cutoff $\bar{\gamma}$ such that every rare bidder with $\alpha > \bar{\gamma}$ submits a score that provides her a certainty equivalent of zero. On the left hand boundary, there are two possibilities. If the certainty equivalent distributions of the frequent and rare groups are sufficiently similar, then the lowest $\alpha$ types of each group will submit the same score as in the standard case. However, if the distributions are too far apart, the bidding functions might be bifurcated as in Hubbard and Kirkegaard (2019). That is, there may be a set of $\alpha$ types from the frequent group that compete against each other at scores too low for rare bidders to be willing to participate. In this case, there are also two different starting scores $s_f < s_r$ such that $\varphi_f(s_f) = \alpha$ and $\varphi_f(s_r) = \alpha$.

To compute equilibria for each auction, we solve a boundary value problem with a shooting algorithm based on Hubbard and Paarsch (2014). To account for bifurcation, we allow for two regions in the ODE solution: a homogeneous region in which only one group of bidders compete against each other, and a heterogeneous region that follows the equations described in this section. We find the beginning of the heterogeneous region by iteratively checking for the first score at which both groups of bidders have a type willing to participate in bidding. While shooting methods are known to be highly sensitive to the step size of their integration method, we found that a modified shooting method with high-step-size Euler integration worked most consistently for our problem. However, as the number of steps required for convergence increases with numerical complexity, we focused on a subset

---

20See Hubbard and Paarsch (2014) for a comprehensive survey.
21This procedure is similar to the check for “active” bidders in Somaini (2020).
of auctions for comparison with our baseline model.

In Table 9, we present a comparison of our main counterfactuals under the baseline model and under the asymmetric model for a sample of auctions in our data. The sample accounts for roughly half of the projects in our dataset and is similar in distribution to projects of the Bridge Reconstruction and Rehabilitation category. Among the projects included, the average $\gamma$ in our baseline (homogeneous) specification is 0.49 (with standard deviation 0.083 and median 0.029). By contrast, the average frequent type $\gamma$ is 0.040 (s.d., 0.065; median, 0.028) and the average rare type $\gamma$ is 0.048 (s.d., 0.048; median, 0.027).

In general, we find that the results between the 1-type and 2-type cases are within a few percentage points of each other. For the median auction in this sample, the 2-type model predicts a risk premium that is 0.4 percentage points higher and a lump sum cost that is 0.2 percentage points lower than the 1-type model. A similar pattern holds at the average and every quartile as well. This suggests that the value of lowering risk by using a scaling format or by reducing uncertainty may be a bit lower if bidders are asymmetric, but that our baseline 1-type model is capturing the overall magnitude of the counterfactual effects well.

<table>
<thead>
<tr>
<th>CF Type</th>
<th>Outcome</th>
<th>Mean</th>
<th>SD</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Risk (Correct q)</td>
<td>% DOT Savings 1-Type</td>
<td>17.8</td>
<td>11.9</td>
<td>10.3</td>
<td>16.9</td>
<td>22.5</td>
</tr>
<tr>
<td>No Risk (Correct q)</td>
<td>% DOT Savings 2-Type</td>
<td>18.2</td>
<td>11.9</td>
<td>10.9</td>
<td>17.3</td>
<td>23.5</td>
</tr>
<tr>
<td>No Risk (Correct q)</td>
<td>$ DOT Savings 1-Type</td>
<td>217,539</td>
<td>200,062</td>
<td>80,474</td>
<td>161,121</td>
<td>293,246</td>
</tr>
<tr>
<td>No Risk (Correct q)</td>
<td>$ DOT Savings 2-Type</td>
<td>225,817</td>
<td>213,274</td>
<td>85,708</td>
<td>169,542</td>
<td>298,545</td>
</tr>
<tr>
<td>Lump Sum (Correct q)</td>
<td>% DOT Savings 1-Type</td>
<td>-90.7</td>
<td>245.8</td>
<td>-77.0</td>
<td>-21.6</td>
<td>4.1</td>
</tr>
<tr>
<td>Lump Sum (Correct q)</td>
<td>% DOT Savings 2-Type</td>
<td>-84.0</td>
<td>218.4</td>
<td>-73.6</td>
<td>-21.4</td>
<td>3.2</td>
</tr>
<tr>
<td>Lump Sum (Correct q)</td>
<td>$ DOT Savings 1-Type</td>
<td>-1,696,121</td>
<td>7,843,739</td>
<td>-929,132</td>
<td>-242,667</td>
<td>13,323</td>
</tr>
<tr>
<td>Lump Sum (Correct q)</td>
<td>$ DOT Savings 2-Type</td>
<td>-1,600,511</td>
<td>6,827,165</td>
<td>-926,175</td>
<td>-222,249</td>
<td>8,240</td>
</tr>
</tbody>
</table>

Table 9: Comparisons of 1-type vs 2-type Counterfactuals for a Sample of Auctions

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22We chose projects to include in the sample on the basis of computational efficiency. This tends to select for projects with lower overall risk aversion and costs, as these projects exhibit more stable numerical behavior at the tails. The total sample is 248 projects. However as we were not able to get convergence on all projects in all auction formats, our comparison includes 237 projects in the no risk counterfactual and 204 projects in the lump sum counterfactual.
D Technical Details

D.1 Econometric Details

Let $b_{t,i,n}^d$ denote the unit bid observed by the econometrician for item $t$, by bidder $i$ in auction $n$. Let $\theta = (\theta_1, \theta_2)$ be the vector of variables that parameterize the model prediction for each bid $b_{t,i,n}^*(\theta)$, as defined by Equation (6). The sub-vector $\theta_1$ refers to parameters estimated in the first stage, as detailed in Appendix D.1.1. The sub-vector $\theta_2$ refers to parameters estimated in the second stage, as detailed in Appendix D.1.2. By Assumption 1, the residual of the optimal bid for each item-bidder-auction tuple with respect to its noisily observed bid: $\nu_{t,i,n} = b_{t,i,n}^d - b_{t,i,n}^*(\theta)$, is distributed identically and independently with a mean of zero across items, bidders and auctions. Furthermore, $\nu_{t,i,n}$ is orthogonal to the identity and characteristics of each item, bidder and auction.$^{23}$

Our estimation procedure treats each auction $n$ as a random sample from some unknown distribution. As such, auctions are exchangeable. Each auction $n$ has an associated set of bidders who participate in the auction, $\mathcal{I}(n)$, as well as an associated set of items that receive bids in the auction, $\mathcal{T}(n)$. $\mathcal{I}(n)$ and $\mathcal{T}(n)$ are characteristics of auction $n$ and so are drawn according to the underlying distribution over auctions themselves. For each bidder $i \in \mathcal{I}(n)$ and item $t \in \mathcal{T}(n)$, our model assigns a unique true bid $b_{t,i,n}^*(\theta)$ at the true parameter vector $\theta$.

Items $t \in \mathcal{T}(n)$ are characterized by a $P \times 1$ vector, $X_{t,n}$, of features. Bidders $i \in \mathcal{I}(n)$ are characterized by a $J \times 1$ vector, $X_{i,n}$, of features. The construction of $X_{t,n}$ and $X_{i,n}$ is discussed in detail in Appendix D.2. Estimation proceeds in two stages. In the first stage, we estimate $\theta_1$, the sub-vector of parameters that governs bidders’ beliefs over ex-post item quantities, using a best-predictor model estimated with Hamiltonian Monte Carlo. In the second stage, we estimate $\theta_2$, which characterizes bidders’ risk aversion and cost types, using a GMM estimator.

D.1.1 First Stage

In the first stage, we use the full dataset of auctions available to us in order to estimate a best-predictor model of expected item quantities conditional on DOT estimates and project-item characteristics, as well as the level of uncertainty that characterizes each projection.

Each observation is an instance of a type of item $t$, being used in an auctioned project $n$. Each observation $(t, n)$ is associated with a vector of item-auction characteristic features $X_{t,n}$,

---

$^{23}$It is not strictly necessary to assume IID across bidders and items. However, allowing for further heterogeneity complicates estimation substantially and so we defer this to a robustness check using Bayesian methods.
the construction of which is discussed in Appendix D.2 below. For simplicity, we employ a linear model for the expected quantity of item \( t \) in auction \( n \), \( \hat{q}_{t,n}^{b} \) as a function of the DOT quantity estimate \( q_{t,n}^{e} \) and \( X_{t,n} \).\(^{24} \) In order to model the level of uncertainty in the projection \( \hat{q}_{t,n}^{b} \), we model the distribution of the quantity model fit residuals (\( q_{t,n}^{e} = q_{t,n}^{b} - \hat{q}_{t,n}^{b} \)) with a lognormal regression function of \( q_{t,n}^{e} \) and \( X_{t,n} \) as well. The full model specification is below. While we could fit this in two stages (first, fit the expected quantity and then fit the distribution of the residuals), we do this jointly using Hamiltonian Monte Carlo (HMC) with the Stan probabilistic programming language.\(^{25} \) We then take the posterior modes of the estimated distributions and use them as point estimates for the second stage.

\[
q_{t,n}^{a} = q_{t,n}^{b} + \eta_{t,n} \text{ where } \eta_{t,n} \sim \mathcal{N}(0, \hat{\sigma}_{t,n}^{2}) \quad (51)
\]

such that

\[
\hat{q}_{t,n}^{b} = \beta_{0,q} q_{t,n}^{e} + \beta_{q} X_{t,n} \text{ and } \hat{\sigma}_{t,n} = \exp(\beta_{0,\sigma} q_{t,n}^{e} + \beta_{\sigma} X_{t,n}). \quad (52)
\]

Denote \( \theta_1 = (\beta_{0,q}, \beta_{q}, \beta_{0,\sigma}, \beta_{\sigma}, \bar{\beta}_{q}, \bar{\beta}_{\sigma}, \bar{\sigma}) \) for the vector of first stage parameters and let \( \hat{\theta}_1 \) be the posterior modes of \( \theta_1 \), produced by the first stage HMC estimation. Thus, \( \hat{\theta}_1 \) specifies, for each item \( t \in \mathcal{T}(n) \) in each auction \( n \), the model estimate of bidders’ predictions for the item’s quantity: \( \hat{q}_{t,n}^{b} \) as well as the variance of that prediction, \( \hat{\sigma}_{t,n}^{2} \).

### D.1.2 Second Stage

Denote \( \theta_2 = (\beta_{0,\gamma}, \beta_{\gamma}, \ldots, \beta_{\gamma}^{J}, \alpha^{1}, \ldots, \alpha^{I}, \beta_{\alpha}^{1}, \ldots, \beta_{\alpha}^{J}) \) for the vector of second stage parameters, where \( I \) is the number of unique firm IDs and \( J \) is the number of auction-bidder features.\(^{26} \)

We estimate \( \theta_2 \) in the second stage, using a GMM framework, evaluated at the first stage estimates \( \hat{\theta}_1 \):

\[
\theta_2 = \arg \min \mathbb{E}_n \left[ g(\theta_2, \hat{\theta}_1)'Wg(\theta_2, \hat{\theta}_1) \right]
\]

where \( g(\theta_2, \hat{\theta}_1) \) is a vector of moments, as a function of \( \theta_2 \), evaluated at the estimates of \( \theta_1 \) obtained in the first stage, and \( W \) is a weighting matrix. We make use of the following 5 types of moments, asymptotic in the number of auctions \( N \). The first type of moment states that the average residual of a unit bid submitted for an item labeled as a “top skew item” by the DOT chief engineer’s office is zero across auctions. There is one such moment. The second type of moment states that the average residual of a unit bid submitted by each

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\(^{24} \)In principle, any statistical model (not necessarily a linear one) would be sound.

\(^{25} \)See Carpenter, Gelman, Hoffman, Lee, Goodrich, Betancourt, Brubaker, Guo, Li, and Riddell (2017) for details on Stan.

\(^{26} \)To simplify notation, we do not distinguish between ‘unique’ bidders—e.g. bidders who appear in 30+ auctions—and rare bidders, whom we group into a single unique bidder ID for the purposes of this econometrics section. For the latter group, we treat all observations of rare bidders as observations of the same single bidder, who may enter a given auction more than once, with a different draw of auction-bidder characteristics, but the same bidder fixed effect determining her efficiency type.
(unique) bidder $i$ is zero across auctions. There are $I$ such moments, where $I$ is the number of unique bidders. The third type of moment states that the average residual on a unit bid submitted in each auction is zero, independent of the auction-bidder characteristics of the bidder submitting the bid. There are $J$ such moments—one for each of the auction-bidder characteristics. Finally, we form moments by interacting the “top skew item” identifier the identifiers for the bidder-level (type 2) and characteristic-level (type 3) moments. Given the propagation of risk across the two steps, we use equal weighting for $W$ and compute standard errors by a two-stage bootstrap procedure.\footnote{Although our GMM system is over-identified we did not find that changing the weighting matrix made a big difference, and so we kept the identity matrix for computational purposes.}

\begin{align*}
m^1_s(\theta_2|\hat{\theta}_1) &= \mathbb{E}_n \left[ \frac{1}{|I(n)|} \sum_{i \in I(n)} \sum_{t \in T(n)} \tilde{\nu}_{t,i,n}(\theta_2|\hat{\theta}_1) \cdot 1_{i \in I(n)} \cdot 1_{t \in T_s} \right] \\
m^2_j(\theta_2|\hat{\theta}_1) &= \mathbb{E}_n \left[ \frac{1}{|T(n)|} \sum_{t \in T(n)} \tilde{\nu}_{t,i,n}(\theta_2|\hat{\theta}_1) \cdot 1_{i \in I(n)} \right] \\
m^3_j(\theta_2|\hat{\theta}_1) &= \mathbb{E}_n \left[ \frac{1}{|I(n)|} \sum_{i \in I(n)} \sum_{t \in T(n)} \tilde{\nu}_{t,i,n}(\theta_2|\hat{\theta}_1) \cdot 1_{i \in I(n)} \cdot X_{i,n}^j \right] \\
m^4_i(\theta_2|\hat{\theta}_1) &= \mathbb{E}_n \left[ \frac{1}{|T_s(n)|} \sum_{t \in T(n)} \tilde{\nu}_{t,i,n}(\theta_2|\hat{\theta}_1) \cdot 1_{i \in I(n)} \cdot 1_{t \in T_s} \right] \\
m^5_j(\theta_2|\hat{\theta}_1) &= \mathbb{E}_n \left[ \frac{1}{|I(n)|} \sum_{i \in I(n)} \sum_{t \in T(n)} \tilde{\nu}_{t,i,n}(\theta_2|\hat{\theta}_1) \cdot 1_{i \in I(n)} \cdot 1_{t \in T_s} \cdot X_{i,n}^j \right]
\end{align*}

For each auction $n$, we denote $I(n)$ as the set of bidders involved in $n$, $T(n)$ as the set of items used in $n$, $T_s$ as the subset of items that were labeled as “top skew items” by the DOT chief engineer’s office, and $T_s(n)$ as the set of “top skew items” in auction $n$. All moments
are formed with respect to the *de-meaned* bid residual:

\[
\tilde{\nu}_{t,i,n}(\theta_2|\hat{\theta}_1) = b_{t,i,n}^d - \alpha_{i,n}(\theta_2) \left( c_{t,n} - \frac{q_{t,n}^e}{\hat{\sigma}_{t,n}^2} \sum_{p \in T(n)} \left[ \frac{(q_{p,n}^e)^2}{\hat{\sigma}_{p,n}^2} \right] \left[ \sum_{p \in T(n)} c_{p,n} q_{p,n}^e \right] \right) - \frac{1}{\gamma_n(\theta_2)} \left( \frac{q_{t,n}^b}{\hat{\sigma}_{t,n}^2} - \frac{q_{t,n}^e}{\hat{\sigma}_{t,n}^2} \sum_{p \in T(n)} \left[ \frac{(q_{p,n}^e)^2}{\hat{\sigma}_{p,n}^2} \right] \left[ \sum_{p \in T(n)} \frac{\hat{q}_{p,n}^b q_{p,n}^e}{\hat{\sigma}_{p,n}^2} \right] \right) - \frac{q_{t,n}^e}{\hat{\sigma}_{t,n}^2} \sum_{p \in T(n)} \left[ \frac{(q_{p,n}^e)^2}{\hat{\sigma}_{p,n}^2} \right] \left[ s_{t,n}^d \right],
\]

where

\[
\alpha_{i,n}(\theta_2) = \alpha_i(\theta_2) + \beta_\alpha(\theta_2) X_{i,n} \quad \text{and} \quad \gamma_n(\theta_2) = \gamma_0(\theta_2) + \beta_\gamma(\theta_2) X_n
\]

The residual terms in the moments are *de-meaned* in the sense that they use the observed score \(s_{i,n}^d\) in the formulation of the optimal bid for \((t, i, n)\), rather than the true optimal score, \(s_{i,n}^*\). That is, since \(s_{i,n}^d\) is composed of noisily observed unit bids, the de-meaned residual \(\tilde{\nu}_{t,i,n}(\theta_2|\hat{\theta}_1)\) omits an unobserved score error term:

\[
\tilde{\nu}_{t,i,n} = \nu_{i,t,n} - \frac{q_{t,n}^e}{\hat{\sigma}_{t,n}^2} \sum_{p \in T(n)} \left[ \frac{(q_{p,n}^e)^2}{\hat{\sigma}_{p,n}^2} \right] \hat{\nu}_{t,n}, \tag{53}
\]

where

\[
\hat{\nu}_{t,n} = - \sum_{t=1}^{T_n} \nu_{i,t,n} q_{t,n}^e. \tag{54}
\]

However, as bid residuals \(\nu_{i,t,n}\) are assumed to be mean zero and independent of auction and item characteristics, \(E_n[\hat{\nu}_{t,n}]\), and the unobserved score error term is mean zero as well. Thus, the use of demeaned bid residuals does not pose a bias for our GMM estimation procedure.

**Estimation Procedure**  To summarize, we estimate our parameters in a two-stage procedure. In the first stage, we estimate the informational parameters that model bidders’ expectations over item quantities and competing scores. In the second stage, we use a two-step optimal GMM estimator to estimate the economic parameters:

1. Estimate \(\hat{\theta}_1 = (\hat{\beta}_{0,q}, \hat{\beta}_q, \hat{\beta}_{0,\sigma}, \hat{\beta}_{\sigma}, \hat{\beta}_s, \hat{\sigma}_s)\) and initialize \(\theta_2\)
2. Solve:

\[
\hat{\theta}_2 = \min_{\theta_2} \left\{ m_1^2(\theta_2|\hat{\theta}_1)^2 + \frac{1}{I} \sum_i m_i^2(\theta_2|\hat{\theta}_1)^2 + \frac{1}{J} \sum_j m_j^2(\theta_2|\hat{\theta}_1)^2 \right. \\
+ \left. \frac{1}{I} \sum_i m_i^4(\theta_2|\hat{\theta}_1)^2 + \frac{1}{J} \sum_j m_j^4(\theta_2|\hat{\theta}_1)^2 \right\}
\]

where \( I \) is the set of unique firm IDs and \( J \) is the number of columns in \( X_{i,n} \). This optimization problem is solved subject to the constraint that \( \alpha_{i,n}(\theta_2) \) be non-negative for every \( i \) and \( n \).\(^{28}\)

We calculate standard errors by a two-step bootstrap procedure. First we take 100 draws from the posterior distribution of quantity distribution in stage 1 of our estimation procedure. Next, we draw 100 auctions at random with replacement from the total set of auctions in our sample, and repeat the step 2 optimization procedure. The confidence intervals presented correspond to the 5th and 95th percentile of the parameter estimates across the bootstrap draws.

**D.2 Projecting Items and Bidder-Auction Pairs onto Characteristic Space**

Our dataset consists of 440 bridge projects with a total of 218,110 unit bid observations. Of these, there are 2,883 unique bidder-project pairs and 29,834 unique item-project pairs. Each auction has an average of 6.55 bidders and 67.8 items. Of these, there are 116 unique bidders and 2,985 unique items (as per the DOT’s internal taxonomy). In order to keep the computational burden of our estimator within a manageable range, while still capturing heterogeneity across bidders and items within and across projects, we project item-project and bidder-project pairs onto characteristic space.

We first build a characteristic-space model of items as follows. The DOT codes each item observation in two ways: a 6-digit item id, and a text description of what the item is. Each item id comprises a hierarchical taxonomy of item classification. That is, the more digits two items have in common (from left to right), the closer the two items are. For example, item 866100 – also known as ”100 Mm Reflect. White Line (Thermoplastic)” – is much closer to item 867100 – ”100 Mm Reflect. Yellow Line (Thermoplastic)” – than it is to item 853100 –

\(^{28}\)This is a computationally efficient approach to impose the theoretical restriction that bidder costs are positive (so that bidders do not gain money from using materials). One could alternatively impose this through an additional moment condition. However, this would add a substantial computational burden as indicators for non-negativity are non-differentiable functions. We provide estimates without the non-negativity constraint as a robustness check. The results do not differ to an economically significant degree.
"Portable Breakaway Barricade Type Iii", and even farther from item 701000 – "Concrete Sidewalk". To leverage the information in both the item ids and the description, we break the ids into digits, and tokenize the item description.\(^{29}\) We then add summary statistics for each item: the relative commonness with which the item is used in projects, the average DOT cost estimate for that item, and dummies that indicate whether or not the item is frequently used in a single unit quantity, and whether the item is often ultimately not used at all.

We create an item-project level characteristic matrix by combining the item characteristic matrix with project-level characteristics: the project category, the identities of the project manager, designer and engineer, the district in which the project is located, the project duration, the number of items in the project spec that the engineer has flagged for us as "commonly skewed", and the share of projects administered by the manager and engineer that over/under-ran.\(^{30}\) The resulting matrix is very high dimensional, and so we project the matrix onto its principle components, and use the first 15.\(^{31}\) In addition, we added 3 stand-alone project features: a dummy variable indicating whether the item is often given a single unit quantity (indicating that its quantity is particularly discrete), the historical share of observations of that item in which it was not used at all, and an indicator for whether or not the item itself is a "commonly skewed" item. The result is the matrix \(X_{t,n}\), used in the estimation in Equation (41).

To estimate the efficiency type \(\alpha_{i,n}\) for each bidder-auction pair, we combine each bidder’s unique firm ID with the matrix of project characteristics described above, and a matrix of project-bidder specific features. As a number of bidders only participate in a few auctions, we combine all bidders who appear in less than 10 auctions in our data set into a single firm ID. This results in 52 unique bidder IDs: 51 unique firms and one aggregate group. For project-bidder characteristics, we compute the bidder’s specialization in each project type—the share of projects of the same type as the current project that the bidder has bid on—the bidder’s capacity—the maximum number of DOT projects that the DOT has ever had open while bidding on another project—and the bidder’s utilization—the share of the bidder’s capacity that is filled when she is bidding on the current project. We also include

\(^{29}\)That is, we split each description up by words, clean them up and remove common “stop” words. Then we create a large dummy matrix in which entry \(i,j\) is 1 if the unique item indexed at \(i\) contains the word indexed by \(j\) in its description. We owe a big thanks to Jim Savage for suggesting this approach.

\(^{30}\)There are 11 items that have been flagged at our request by the cheif engineer: 120100: Unclassified Excavation; 129600: Bridge Pavement Excavation; 220000: Drainage Structure Adjusted; 450900: Contractor Quality Control; 464000: Bitumen For Tack Coat; 472000: Hot Mix Asphalt For Miscellaneous Work; 624100: Steel Thrie Beam Highway Guard (Double Faced); 851000: Safety Controls For Construction Operations (Traffic Cones For Traffic Management); 853200: Temporary Concrete Barrier; 853403: Movable Impact Attenuator; 853800: Temporary Illumination For Work Zone (Temporary Illumination For Night Work)

\(^{31}\)We have tried replicating this using more/less principle components and the results are very stable.
dummies for whether or not the bidder is a *fringe* bidder, and whether or not the bidder’s headquarters is located in the same district as the project at hand.\textsuperscript{32} Our $X_{i,n}$ matrix has a total of 14 columns, and so we have a total of 66 efficiency-type parameters to identify. We use $X_{i,n}$ and the unique bidder ideas to model $\alpha_{i,n}^t$ in equation D.1.2.

Finally, we make use of a project-level characteristic matrix $X_n$ in our counterfactuals, in order to parametrize the distribution of efficiency types in each auction. In principle, we could use the bidder-auction matrix $X_{i,n}$ here. However, this would require each bidder to know the identities of her competitors. For the purpose of our main counterfactuals, we focus on the simpler case in which the distribution of scores is homogenous across the bidders participating in a given auction. Therefore, we construct $X_n$ by taking an average of $X_n$ with respect to the bidders in auction $n$.

\begin{footnote}
\textsuperscript{32}We define “fringe” similarly to BHT, as a firm that receives less than 1% of the total funds spent by the DOT on projects within the same project type as the auction being considered, within the scope of our dataset.
\end{footnote}
## Estimation Results Tables

### First Stage Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Rhat</th>
<th>n_eff</th>
<th>mean</th>
<th>sd</th>
<th>2.5%</th>
<th>50%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
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<td>4000</td>
<td>-0.67</td>
<td>0.00</td>
<td>-0.67</td>
<td>-0.67</td>
<td>-0.66</td>
</tr>
<tr>
<td>$\beta_1$</td>
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<td>0.01</td>
<td>-0.06</td>
<td>-0.05</td>
<td>-0.04</td>
</tr>
<tr>
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<td>2120</td>
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<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
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<td>3275</td>
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<td>-0.02</td>
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<td>0.02</td>
<td>0.03</td>
</tr>
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<td>0.03</td>
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<td>0.04</td>
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<td>-0.01</td>
<td>0.00</td>
<td>0.01</td>
</tr>
<tr>
<td>$\beta_9$</td>
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<td>-0.03</td>
<td>-0.02</td>
<td>-0.01</td>
</tr>
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<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
</tr>
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<td>$\beta_{11}$</td>
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<td>-0.03</td>
<td>-0.02</td>
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<td>0.02</td>
<td>0.03</td>
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<td>0.04</td>
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<td>0.00</td>
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<td>0.00</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
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<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
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<td>-0.02</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\beta_2,q$</td>
<td>1.00</td>
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<td>-0.02</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
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<td>-0.04</td>
<td>-0.03</td>
<td>-0.02</td>
</tr>
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<td>$\beta_4,q$</td>
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<td>$\beta_5,q$</td>
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<td>-0.03</td>
<td>-0.02</td>
<td>-0.01</td>
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<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
</tr>
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<td>0.02</td>
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<td>$\beta_8,q$</td>
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<td>-0.02</td>
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<td>-0.00</td>
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</tr>
<tr>
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<td>-0.00</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$\beta_{14},q$</td>
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<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>$\beta_{15},q$</td>
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<td>0.00</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>$\beta_{16},q$</td>
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<td>0.01</td>
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<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
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<tr>
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<td>-0.18</td>
<td>-0.17</td>
</tr>
<tr>
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<td>0.00</td>
<td>-0.02</td>
<td>-0.01</td>
<td>-0.00</td>
</tr>
</tbody>
</table>

Table 10: First Stage Parameter Estimates
Second Stage Parameter Estimates

We obtain standard errors and confidence bounds through a two-step bootstrapping procedure. We first take 100 draws from the posterior distribution of the first stage model. Then for each first stage draw, we perform 100 bootstrap iterations of the second stage estimation procedure. In each iteration, we redraw 438 auctions at random with replacement and reestimate our second stage GMM model. Table 11 presents the resulting 95% confidence interval for the headline estimates in Tables 3, 4, and 5, as well as their standard deviations both across all draws, and within the 95% confidence interval.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SD</th>
<th>SD within CI</th>
<th>2.5%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean α</td>
<td>0.943</td>
<td>0.087</td>
<td>0.076</td>
<td>0.858</td>
<td>1.180</td>
</tr>
<tr>
<td>25% α</td>
<td>0.837</td>
<td>0.097</td>
<td>0.085</td>
<td>0.683</td>
<td>1.051</td>
</tr>
<tr>
<td>50% α</td>
<td>0.975</td>
<td>0.089</td>
<td>0.078</td>
<td>0.859</td>
<td>1.202</td>
</tr>
<tr>
<td>75% α</td>
<td>1.099</td>
<td>0.092</td>
<td>0.079</td>
<td>1.025</td>
<td>1.373</td>
</tr>
<tr>
<td>Mean γ</td>
<td>0.071</td>
<td>0.046</td>
<td>0.028</td>
<td>0.069</td>
<td>0.214</td>
</tr>
<tr>
<td>25% γ</td>
<td>0.024</td>
<td>0.015</td>
<td>0.011</td>
<td>0.021</td>
<td>0.078</td>
</tr>
<tr>
<td>50% γ</td>
<td>0.044</td>
<td>0.064</td>
<td>0.020</td>
<td>0.033</td>
<td>0.139</td>
</tr>
<tr>
<td>75% γ</td>
<td>0.073</td>
<td>0.075</td>
<td>0.036</td>
<td>0.053</td>
<td>0.252</td>
</tr>
<tr>
<td>Mean Markup</td>
<td>0.4</td>
<td>0.143</td>
<td>0.118</td>
<td>0.093</td>
<td>0.648</td>
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<td>25% Markup</td>
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<td>0.067</td>
<td>0.058</td>
<td>-0.201</td>
<td>0.053</td>
</tr>
<tr>
<td>50% Markup</td>
<td>0.17</td>
<td>0.097</td>
<td>0.082</td>
<td>-0.035</td>
<td>0.315</td>
</tr>
<tr>
<td>75% Markup</td>
<td>0.51</td>
<td>0.159</td>
<td>0.134</td>
<td>0.170</td>
<td>0.756</td>
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Table 11: Second Stage Bootstrap Errors and Quantiles

First Stage Model Fit

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<th>Dependent variable:</th>
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<tr>
<td>Actual Quantity</td>
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<tr>
<td>Predicted Quantity</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Constant</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>R²</td>
</tr>
</tbody>
</table>

Table 12: Regression report for Figure 6

Figure 6: A bin scatter of actual quantities vs model predictions
Second Stage Model Fit

Figure 7: A scatter plot of actual quantities vs model predictions. Note: Unit bids are scaled so as to standardize quantities so exact dollar values are not representative.

![Scatter plot of actual quantities vs model predictions](image)

Table 13: Regression report for figure 7

<table>
<thead>
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<th>Dependent variable: Data Bid</th>
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<tr>
<td>Predicted Bid</td>
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<tr>
<td>Constant</td>
</tr>
<tr>
<td>Observations</td>
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<tr>
<td>R²</td>
</tr>
</tbody>
</table>

*Note:* *p<0.1; **p<0.05; ***p<0.01

Figure 8: Fit plots for bids and scores with the 45-degree line, dashed in red, for reference.

(a) Quantile-Quantile plot of predicted bids against data bids. Quantiles are presented at the 0.0001 level and truncated at the top and bottom 0.01% for clarity.

(b) A scatter plot of actual winning scores against the winning scores predicted by our equilibrium simulation at the estimated parameters.
F Counterfactual Results Tables

We report the summary statistics for the counterfactual results reported in Section 8.\textsuperscript{33}

<table>
<thead>
<tr>
<th>CF Type</th>
<th>Outcome</th>
<th>Mean</th>
<th>SD</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
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<td><strong>Main CFs</strong></td>
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<tr>
<td>No Risk (Correct q)</td>
<td>% DOT Savings</td>
<td>14.9</td>
<td>10.1</td>
<td>7.0</td>
<td>13.6</td>
<td>20.2</td>
</tr>
<tr>
<td>No Risk (Correct q)</td>
<td>$ DOT Savings</td>
<td>229,076.1</td>
<td>232,988.9</td>
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<td>289,447.9</td>
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<tr>
<td>No Risk (Estimated q)</td>
<td>% DOT Savings</td>
<td>0.8</td>
<td>5.0</td>
<td>-0.9</td>
<td>0.3</td>
<td>1.7</td>
</tr>
<tr>
<td>No Risk (Estimated q)</td>
<td>$ DOT Savings</td>
<td>2,856.4</td>
<td>33,736.9</td>
<td>-13,512.0</td>
<td>3,454.2</td>
<td>20,697.9</td>
</tr>
<tr>
<td>Lump Sum (Correct q)</td>
<td>% DOT Savings</td>
<td>-153.3</td>
<td>244.2</td>
<td>-180.4</td>
<td>-68.4</td>
<td>-12.7</td>
</tr>
<tr>
<td>Lump Sum (Correct q)</td>
<td>$ DOT Savings</td>
<td>-4,080,494.6</td>
<td>11,933,907.5</td>
<td>-2,990,392.3</td>
<td>-953,181.0</td>
<td>-117,166.4</td>
</tr>
<tr>
<td>Lump Sum (Estimated q)</td>
<td>% DOT Savings</td>
<td>-199.5</td>
<td>309</td>
<td>-228.1</td>
<td>-89.8</td>
<td>-36.6</td>
</tr>
<tr>
<td>Lump Sum (Estimated q)</td>
<td>$ DOT Savings</td>
<td>-5,013,135.9</td>
<td>12,808,625</td>
<td>-3,153,769.6</td>
<td>-1,004,313.8</td>
<td>-259,547.2</td>
</tr>
<tr>
<td>25% Min Bid (Correct q)</td>
<td>% DOT Savings</td>
<td>3.8</td>
<td>4.4</td>
<td>0.7</td>
<td>2.2</td>
<td>5.0</td>
</tr>
<tr>
<td>25% Min Bid (Correct q)</td>
<td>$ DOT Savings</td>
<td>43,510.6</td>
<td>51,389.6</td>
<td>12,435.8</td>
<td>27,463.3</td>
<td>54,824.8</td>
</tr>
<tr>
<td>25% Min Bid (Estimated q)</td>
<td>% DOT Savings</td>
<td>0.2</td>
<td>0.9</td>
<td>-0.1</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>25% Min Bid (Estimated q)</td>
<td>$ DOT Savings</td>
<td>1,675.7</td>
<td>7,050.7</td>
<td>-1,716.6</td>
<td>1,339.6</td>
<td>4,850.5</td>
</tr>
<tr>
<td><strong>Extensions (Correct q)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lump Sum w 2:1 Negotation</td>
<td>% DOT Savings</td>
<td>-13.3</td>
<td>21.2</td>
<td>-20.9</td>
<td>-7.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Lump Sum w 2:1 Negotation</td>
<td>$ DOT Savings</td>
<td>-317,457.8</td>
<td>758,581.6</td>
<td>-326,345.8</td>
<td>-105,941.5</td>
<td>5,378.3</td>
</tr>
<tr>
<td>Lump Sum w 50/50 Negotiation</td>
<td>% DOT Savings</td>
<td>-5.8</td>
<td>11.2</td>
<td>-9.7</td>
<td>-2.7</td>
<td>1.6</td>
</tr>
<tr>
<td>Lump Sum w 50/50 Negotiation</td>
<td>$ DOT Savings</td>
<td>-137,944.8</td>
<td>355,159.6</td>
<td>-143,863.7</td>
<td>-36,933.3</td>
<td>12,305.6</td>
</tr>
<tr>
<td>No Risk w 10% EWO</td>
<td>% DOT Savings</td>
<td>14.9</td>
<td>10.1</td>
<td>7.0</td>
<td>13.5</td>
<td>19.9</td>
</tr>
<tr>
<td>No Risk w 10% EWO</td>
<td>$ DOT Savings</td>
<td>229,334.8</td>
<td>232,371.6</td>
<td>91,224.9</td>
<td>167,285.8</td>
<td>286,306.3</td>
</tr>
<tr>
<td>Lump Sum w 10% EWO</td>
<td>% DOT Savings</td>
<td>-149.5</td>
<td>242.9</td>
<td>-171.3</td>
<td>-66.5</td>
<td>-9.7</td>
</tr>
<tr>
<td>Lump Sum w 10% EWO</td>
<td>$ DOT Savings</td>
<td>-4,028,974.8</td>
<td>11,902,922.2</td>
<td>-2,958,677.6</td>
<td>-919,617.2</td>
<td>-71,408.4</td>
</tr>
<tr>
<td>No Risk w 50% EWO</td>
<td>% DOT Savings</td>
<td>16.2</td>
<td>11.9</td>
<td>7.7</td>
<td>13.9</td>
<td>21.0</td>
</tr>
<tr>
<td>No Risk w 50% EWO</td>
<td>$ DOT Savings</td>
<td>238,232.7</td>
<td>228,927.4</td>
<td>103,088.5</td>
<td>175,103.4</td>
<td>289,667.2</td>
</tr>
<tr>
<td>Lump Sum w 50% EWO</td>
<td>% DOT Savings</td>
<td>-144.1</td>
<td>270.3</td>
<td>-162.3</td>
<td>-55.1</td>
<td>-0.9</td>
</tr>
<tr>
<td>Lump Sum w 50% EWO</td>
<td>$ DOT Savings</td>
<td>-4,739,573.6</td>
<td>12,847,960.9</td>
<td>-2,799,766.7</td>
<td>-745,151.8</td>
<td>-10,183.3</td>
</tr>
<tr>
<td>No Risk w 10% MH</td>
<td>% DOT Savings</td>
<td>17.8</td>
<td>9.4</td>
<td>10.4</td>
<td>16.4</td>
<td>22.1</td>
</tr>
<tr>
<td>No Risk w 10% MH</td>
<td>$ DOT Savings</td>
<td>302,863.2</td>
<td>286,056.1</td>
<td>132,551.1</td>
<td>219,833.0</td>
<td>364,075.2</td>
</tr>
<tr>
<td>Lump Sum w 10% MH</td>
<td>% DOT Savings</td>
<td>-148.1</td>
<td>245.4</td>
<td>-170.5</td>
<td>-61.5</td>
<td>-9.6</td>
</tr>
<tr>
<td>Lump Sum w 10% MH</td>
<td>$ DOT Savings</td>
<td>-4,880,186.1</td>
<td>12,844,673.5</td>
<td>-2,911,186.1</td>
<td>-905,680.6</td>
<td>-89,771.8</td>
</tr>
<tr>
<td>No Risk w 50% MH</td>
<td>% DOT Savings</td>
<td>27.9</td>
<td>9.4</td>
<td>21.9</td>
<td>26.7</td>
<td>31.7</td>
</tr>
<tr>
<td>No Risk w 50% MH</td>
<td>$ DOT Savings</td>
<td>618,390.1</td>
<td>619,067.2</td>
<td>244,719.8</td>
<td>443,263.4</td>
<td>723,710.2</td>
</tr>
<tr>
<td>Lump Sum w 50% MH</td>
<td>% DOT Savings</td>
<td>-170.7</td>
<td>673.6</td>
<td>-156.6</td>
<td>-51.4</td>
<td>4.7</td>
</tr>
<tr>
<td>Lump Sum w 50% MH</td>
<td>$ DOT Savings</td>
<td>-5,350,835.9</td>
<td>14,020,751.0</td>
<td>-3,286,206.9</td>
<td>-843,583.5</td>
<td>21,659.8</td>
</tr>
</tbody>
</table>

\textsuperscript{33}In each case, results are truncated at the top and bottom 1% to exclude extreme outliers from the mean/SD calculations. These results exclude several projects in each case, for which the ODE solvers were unable to converge under the “standard” setup. Most of these projects are “extremal” in the sense that the imputed lower bound on the \( \alpha \) type distribution is below the truncation point of 0.5. We were able to get almost all of these to converge by changing the truncation point to 0.25. To maintain a clear comparison across projects, we exclude these from the main results. Including them does not make a significant difference and we are happy to report the full results in whatever way is preferred. Our main results focus on the correct \( q \) case as this limits the number of moving parts and facilitates exposition.
G Robustness Checks

Alternative Specifications

(a) Replication of Figure 3a when the top two bidders’ scores are within 10% of each other. (b) Replication of Figure 4a, without controlling for $\%\Delta q_t$.

Figure 9

Bid Level-Weighted Bins

We replicate the main graphs from Section 4, weighting the dots by the average bid levels in the bins that they represent. This demonstrates that outlier dots are generally relatively small, minor items, and that overestimated items are not rare.

(a) Replication of Figure 3a with weighted bins. (b) Replication of Figure 4a with weighted bins.

Figure 10
A Supplemental Appendix (Not for Publication)

A.1 Additional Tables and Figures

Distribution of Projects by Year in Our Data

<table>
<thead>
<tr>
<th>Year</th>
<th>Num Projects</th>
<th>Percent</th>
<th>Cumul Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1998</td>
<td>0.227</td>
<td>0.227</td>
</tr>
<tr>
<td>2</td>
<td>1999</td>
<td>1.136</td>
<td>1.364</td>
</tr>
<tr>
<td>3</td>
<td>2000</td>
<td>1.136</td>
<td>2.500</td>
</tr>
<tr>
<td>4</td>
<td>2001</td>
<td>4.545</td>
<td>7.045</td>
</tr>
<tr>
<td>5</td>
<td>2002</td>
<td>6.136</td>
<td>13.182</td>
</tr>
<tr>
<td>6</td>
<td>2003</td>
<td>5.909</td>
<td>19.091</td>
</tr>
<tr>
<td>7</td>
<td>2004</td>
<td>5.682</td>
<td>24.773</td>
</tr>
<tr>
<td>8</td>
<td>2005</td>
<td>8.409</td>
<td>33.182</td>
</tr>
<tr>
<td>9</td>
<td>2006</td>
<td>4.773</td>
<td>37.955</td>
</tr>
<tr>
<td>10</td>
<td>2007</td>
<td>7.273</td>
<td>45.227</td>
</tr>
<tr>
<td>11</td>
<td>2008</td>
<td>12.045</td>
<td>57.273</td>
</tr>
<tr>
<td>12</td>
<td>2009</td>
<td>10.455</td>
<td>67.727</td>
</tr>
<tr>
<td>13</td>
<td>2010</td>
<td>13.864</td>
<td>81.591</td>
</tr>
<tr>
<td>14</td>
<td>2011</td>
<td>7.273</td>
<td>88.864</td>
</tr>
<tr>
<td>15</td>
<td>2012</td>
<td>5.455</td>
<td>94.318</td>
</tr>
<tr>
<td>16</td>
<td>2013</td>
<td>4.318</td>
<td>98.636</td>
</tr>
<tr>
<td>17</td>
<td>2014</td>
<td>1.364</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 14: Distribution of projects by year in our data

Most Common Firms

<table>
<thead>
<tr>
<th>Bidder Name</th>
<th># Employees</th>
<th># Auctions Bid</th>
<th># Auctions Won</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIG Corporation</td>
<td>80</td>
<td>297</td>
<td>38</td>
</tr>
<tr>
<td>Northern Constr Services LLC</td>
<td>80</td>
<td>286</td>
<td>26</td>
</tr>
<tr>
<td>SPS New England Inc</td>
<td>75</td>
<td>210</td>
<td>58</td>
</tr>
<tr>
<td>ET&amp;L Corp</td>
<td>1</td>
<td>201</td>
<td>26</td>
</tr>
<tr>
<td>B&amp;E Construction Corp</td>
<td>9</td>
<td>118</td>
<td>16</td>
</tr>
<tr>
<td>NEL Corporation</td>
<td>68</td>
<td>116</td>
<td>36</td>
</tr>
<tr>
<td>Construction Dynamics Inc</td>
<td>22</td>
<td>113</td>
<td>10</td>
</tr>
<tr>
<td>S&amp;R Corporation</td>
<td>20</td>
<td>111</td>
<td>16</td>
</tr>
<tr>
<td>New England Infrastructure</td>
<td>35</td>
<td>95</td>
<td>6</td>
</tr>
<tr>
<td>James A Gross Inc</td>
<td>7</td>
<td>78</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 15: All 24 most common firms in our sample are privately owned, and so there is no publicly available, verifiable information on their revenues or expenses. The numbers of employees presented here were drawn from Manta, an online directory of small businesses, and cross-referenced with LinkedIn, on which a subset of these firms list a range of their employee counts, as of November 2018. Note that there is some ambiguity as to who “counts” as an employee, as such firms often hire additional construction laborers on a project-by-project basis. The “family owned” label is drawn from the firms’ self-descriptions on their websites.
Comparison of Model Estimates with Different Levels of γ Heterogeneity

The main analysis in our paper considers a model in which risk aversion parameters γ can vary across auctions to account for differences in market conditions and the types of firms that may participate in different types of auctions. Our estimation procedure is able to accommodate other models as well, trading off power in our limited sample of auctions against flexibility in capturing heterogeneity in the data. Below, we present a summary table of how cost and risk aversion parameter estimates vary across four models: (1) a model with one γ across all bidders and auctions; (2) our baseline model with one γ type per auction; (3) the 2-type model presented in Appendix C.2; (4) a model in which every bidder-auction pair has its own γ parameter as a function of observables. In order to focus the comparison on bidders who are competitive in the most heterogeneous case, the estimate summaries below cover the winning bidder in each auction, in particular.

### Table 16: Aggregate Summary Statistics of Winners’ γ Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model</th>
<th>Mean</th>
<th>SD</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>Common Type</td>
<td>0.044</td>
<td>0.000</td>
<td>0.044</td>
<td>0.044</td>
<td>0.044</td>
</tr>
<tr>
<td>γ</td>
<td>1 Type/Auction</td>
<td>0.080</td>
<td>0.150</td>
<td>0.024</td>
<td>0.043</td>
<td>0.073</td>
</tr>
<tr>
<td>γ</td>
<td>2 Types/Auction</td>
<td>0.070</td>
<td>0.118</td>
<td>0.024</td>
<td>0.045</td>
<td>0.070</td>
</tr>
<tr>
<td>γ</td>
<td>N Types/Auction</td>
<td>0.129</td>
<td>0.169</td>
<td>0.055</td>
<td>0.079</td>
<td>0.133</td>
</tr>
</tbody>
</table>

### Table 17: Aggregate Summary Statistics of Winners’ α Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model</th>
<th>Mean</th>
<th>SD</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Common Type</td>
<td>1.003</td>
<td>0.326</td>
<td>0.820</td>
<td>0.963</td>
<td>1.162</td>
</tr>
<tr>
<td>α</td>
<td>1 Type/Auction</td>
<td>0.912</td>
<td>0.309</td>
<td>0.800</td>
<td>0.979</td>
<td>1.109</td>
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<tr>
<td>α</td>
<td>2 Types/Auction</td>
<td>0.917</td>
<td>0.307</td>
<td>0.809</td>
<td>0.993</td>
<td>1.113</td>
</tr>
<tr>
<td>α</td>
<td>N Types/Auction</td>
<td>1.065</td>
<td>0.345</td>
<td>0.996</td>
<td>1.149</td>
<td>1.264</td>
</tr>
</tbody>
</table>

A.2 Illustrative Example

Consider the following simple example of infrastructure procurement bidding. Two bidders compete for a project that requires two types of inputs to complete: concrete and traffic cones. MassDOT (“the DOT” for short) estimates that 10 tons of concrete and 20 traffic cones will be necessary to complete the project. Upon inspection, the bidders determine that the actual quantities of each item that will be used – random variables that we will denote $q^c_a$ and $q^r_a$ for concrete and traffic cones, respectively – are normally distributed with means $\mathbb{E}[q^c_a] = 12$ and $\mathbb{E}[q^r_a] = 16$ and variances $\sigma^2_c = 2$ and $\sigma^2_r = 1$.\(^{34}\) We assume that the actual quantities are exogenous to the bidding process,

\(^{34}\)As we discuss in section 5, we assume that the distributions of $q^c_a$ and $q^r_a$ are independent conditional on available information regarding the auction. This assumption, as well as the assumption that the quantity
and do not depend on who wins the auction in any way. Furthermore, we will assume that the bidders’ expectations are identical across both bidders.35

The bidders differ in their private costs for implementing the project. They have access to the same vendors for the raw materials, but differ in the cost of storing and transporting the materials to the site of construction as well as the cost of labor, depending on the site’s location, the state of their caseload at the time and firm-level idiosyncrasies. We therefore describe each bidder’s cost as a multiplicative factor \( \alpha \) of market-rate cost estimate for each item: \( c_c = $8/\text{ton} \) for each ton of Concrete and \( c_r = $12/\text{pack} \) for each pack of 100 traffic cones. Each bidder \( i \) knows her own type \( \alpha^i \) at the time of bidding, as well as the distribution (but not realization) of her opponent’s type.

To participate in the auction, each bidder \( i \) submits a unit bid for each of the items: \( b^i_c \) and \( b^i_r \). The winner of the auction is then chosen on the basis of her score: the sum of her unit bids multiplied by the DOT’s quantity estimates:

\[
s^i = 10b^i_c + 20b^i_r.
\]

Once a winner is selected, she will implement the project and earn net profits of her unit bids, less the unit costs of each item, multiplied by the realized quantities of each item that are ultimately used. At the time of bidding, these quantities are unrealized samples of random variables.

Bidders are endowed with a standard CARA utility function over their earnings from the project with a common constant coefficient of absolute risk aversion \( \gamma \):

\[
u(\pi) = 1 - \exp(-\gamma \pi).
\]

Bidders are exposed to two sources of risk: (1) uncertainty over winning the auction; (2) uncertainty over the profits that they would earn at the realized ex-post quantity of each item.

The profit \( \pi \) that bidder \( i \) earns is either 0, if she loses the auction, or

\[
\pi(b^i, \alpha^i, c, q^a) = q^a_c \cdot (b^i_c - \alpha^i c_c) + q^a_r \cdot (b^i_r - \alpha^i c_r),
\]

if she wins the auction. Bidder \( i \)'s expected utility at the time of the auction is therefore given by:

\[
\mathbb{E}[u(\pi(b^i, \alpha^i, c, q^a))] = \left(1 - \mathbb{E}_{q^a} \left[\exp\left(-\gamma \cdot \pi(b^i, \alpha^i, c, q^a)\right)\right]\right) \times \left(\Pr\{s^i < s^j\}\right).
\]

That is, bidder \( i \)'s expected utility from submitting a set of bids \( b^i_c \) and \( b^i_r \) is the product of the
distributions are not truncated at 0 (so that quantities cannot be negative) are made for the purpose of computational traceability in our structural model. If item quantities are correlated, bidders’ risk exposure is higher, and so our results can be seen as a conservative estimate of this case.35These assumptions align with the characterization of highway and bridge projects in practice: the projects are highly standardized and all decisions regarding quantity changes must be approved by an on-site DOT official, thereby limiting contractors’ ability to influence ex-post quantities.
utility that she expects to get (given those bids) if she were to win the auction, and the probability that she will win the auction at those bids. The expectation of utility conditional on winning is with respect to the realizations of the item quantities \( q^c_a \) and \( q^r_a \), entirely.

As the ex-post quantities are distributed as independent Gaussians, the expected utility term above can be rewritten in terms of the certainty equivalent of bidder \( i \)'s profits conditional on winning:

\[
1 - \exp\left(-\gamma \cdot \text{CE}(b^i, \alpha^i, c, q^a)\right),
\]

where the certainty equivalent of profits \( \text{CE}(b^i, \alpha^i, c, q^a) \) is given by:

\[
\text{Expection of Profits} - \frac{\gamma \sigma^2_c}{2} \cdot (b^i_c - \alpha^i c_c)^2 + \frac{\gamma \sigma^2_r}{2} \cdot (b^i_r - \alpha^i c_r)^2.
\]

Furthermore, as we discuss in Section 5, the optimal selection of bids for each bidder \( i \) can be described as the solution to a two-stage problem:

**Inner:** For each possible score \( s \), choose the bids \( b_c^i \) and \( b_r^i \) that maximize \( \text{CE}(\{b_c^i, b_r^i\}, \alpha^i, c, q^a) \), subject to the score constraint: \( 10b_c^i + 20b_r^i = s \).

**Outer:** Choose the score \( s^*(\alpha^i) \) that maximizes expected utility \( \mathbb{E}[u(\pi(b^i(s), \alpha^i))] \), where \( b^i(s) \) is the solution to the inner step, evaluated at \( s \).

That is, at every possible score that bidder \( i \) might consider, she chooses the bids that sum to \( s \) for the purpose of the DOT's evaluation of who will win the auction, and maximize her certainty equivalent of profits conditional on winning. She then chooses the score that maximizes her total expected utility.

To see how this decision process can generate bids that appear mathematically unbalanced, suppose, for example, that the common CARA coefficient is \( \gamma = 0.05 \), and consider a bidder in this auction who has type \( \alpha^i = 1.5 \).\(^{37}\) Suppose, furthermore, that the bidder has decided to submit a total score of \( $500 \). There are a number of ways in which the bidder could construct a score of \( $500 \). For instance, she could bid her cost on concrete, \( b^i_c = $12 \), and a dollar mark-up on traffic cones: \( b^i_r = (\$500 - $12 \times 10)/20 = $19 \). Alternatively, she could bid her cost on traffic cones, \( b^i_r = $18 \), and a two-dollar mark-up on traffic cones: \( b^i_c = (\$500 - $18 \times 20)/10 = $14 \). Both of these bids would result in the same score, and so give the bidder the same chances of winning the auction. However, they yield very different expected utilities to the bidder. Plugging each set of bids into equation (55), we find that the first set of bids produces a certainty equivalent of:

\[
12 \times ($0) + 16 \times ($1) - \frac{0.05 \times 2}{2} \times ($0)^2 - \frac{0.05 \times 1}{2} \times ($1)^2 = $15.98,
\]

\(^{36}\)See section 5 and the appendix for a detailed derivation.

\(^{37}\)That is, for each ton of concrete that will be used will cost, the bidder incur a cost of \( \alpha^i \times c_c = 1.5 \times $8 = $12 \), and for each pack of traffic cones that will be used, she will incur a cost of \( \alpha^i \times c_r = 1.5 \times $12 = $18 \).
whereas the second set of bids produces a certainty equivalent of

\[ 12 \times ($2) + 16 \times ($0) - \frac{0.05 \times 2}{2} \times ($2)^2 - \frac{0.05 \times 1}{2} \times ($0)^2 = $23.80. \]

In fact, further inspection shows that the optimal bids giving a score of $500 are \( b^i_c = $47.78 \) and \( b^i_r = $1.12, \) yielding a certainty equivalent of $87.98. The intuition for this is precisely that described by Athey and Levin (2001), and the contractors cited by Stark (1974): the bidder predicts that concrete will over-run in quantity – she predicts that 12 tons will be used, whereas the DOT estimated only 10 – and that traffic cones will under-run – she predicts that 16 will be used, rather than the DOT’s estimate of 20. When the variance terms aren’t too large (relatively), the interpretation is quite simple: every additional dollar bid on concrete is worth approximately 12/10 in expectation, whereas every additional dollar bid on traffic cones is worth only 16/20.

However, the incentive to bid higher on items projected to over-run is dampened when the variance term is relatively large. This can arise when the coefficient of risk aversion is relatively high or when the variance of an item’s ex-post quantity distribution is high. More generally, as demonstrated in equation (55), the certainty equivalent of profits is increasing in the expected quantity of each item, \( \mathbb{E}[q^a_c] \) and \( \mathbb{E}[q^a_r], \) but decreasing in the variance of each item \( \sigma^2_c \) and \( \sigma^2_r. \)

![Figure 11](image1.png)

(a) Score = $500

(b) Score = $1000

Figure 11: Certainty equivalent as a function of her unit bid on traffic cones, for the example bidder submitting a score of $500 or $1,000.

Moreover, the extent of bid skewing can depend on the level of competition in the auction. Figure 11 plots the bidder’s certainty equivalent as a function of her unit bid on traffic cones when she chooses to submit a total score of (a) $500, and when she chooses to submit a score of (b) $1,000. In the first case, the bid that optimizes the certainty equivalent is very small, \( b^i_r = $1.12. \) In the second case, however, the optimal bid is much higher at \( b^i_r = $23.33. \) The reason for this is that a low bid on traffic cones implies a high bid on concrete. A high markup on concrete decreases the bidder’s certainty equivalent at a quadratic rate. Thus, as the score gets higher, there is more of an incentive to spread markups across items, rather than bidding very high on select items, and very low on others.
A.3 Bid Skewing in Equilibrium

As we discuss in Section 5, the auction game described above has a unique Bayes Nash Equilibrium. This equilibrium is characterized following the two-stage procedure described in Appendix A: (1) given an equilibrium score \( s(\alpha) \), each bidder of type \( \alpha \) submits the vector of unit bids that maximizes her certainty equivalent conditional on winning, and sums to \( s(\alpha) \); (2) The equilibrium score is chosen optimally, such that there does not exist a type \( \alpha \) and an alternative score \( \tilde{s} \), so that a bidder of type \( \alpha \) can attain a higher expected utility with the score \( \tilde{s} \) than with \( s(\alpha) \).

The optimal selection of bids given an equilibrium score depends on the bidders’ expectations over ex-post quantities and the DOT’s posted estimates, as well as on the coefficient of risk aversion and the level of uncertainty in the bidders’ expectations. High over-runs cause bidders to produce more heavily skewed bids, whereas high risk aversion and high levels of uncertainty push bidders to produce more balanced bids.

In addition to influencing the relative skewness of bids, these factors also have a level effect on bidder utility. Higher expectations of ex-post quantities raise the certainty equivalent conditional on winning for every bidder. Higher levels of uncertainty (and a higher degree of risk aversion), however, induce a cost for bidders that lowers the certainty equivalent. Consequently, higher levels of uncertainty lower the value of participating for every bidder and result in less aggressive bidding behavior, and higher costs to the DOT in equilibrium.

To demonstrate this, we plot the equilibrium score, unit-bid distribution and ex-post revenue for every bidder type \( \alpha \) in our example. To illustrate the effects of risk and risk aversion on bidder behavior and DOT costs, we compare the equilibria in four cases. First, we compute the equilibrium in our example when bidders are risk averse with CARA coefficient \( \gamma = 0.05 \), and when bidders are risk neutral (e.g. \( \gamma = 0 \)). To hone in on the effects of risk in particular, and not mis-estimation, we will assume that the bidders’ expectations of ex-post quantities are perfectly correct (e.g. the realization of \( q_\alpha^a \) is equal to \( \mathbb{E}[q_\alpha^a] \), although the bidders do not know this ex-ante, and still assume their estimates are noisy with Gaussian error).

Next, we compute the equilibrium in each case under the counterfactual in which uncertainty regarding quantities is eliminated. In particular, we consider a setting in which the DOT is able to discern the precise quantities that will be used, and advertise the project with the ex-post quantities, rather than imprecise estimates. The DOT’s accuracy is common knowledge, and so upon seeing the DOT numbers in this counterfactual, the bidders are certain of what the ex-post quantities will be (e.g. \( \sigma^2_c = \sigma^2_r = 0 \)).

<table>
<thead>
<tr>
<th></th>
<th>Risk Neutral Bidders</th>
<th>Risk Averse Bidders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noisy Quantity Estimates</td>
<td>$326.76</td>
<td>$317.32</td>
</tr>
<tr>
<td>Perfect Quantity Estimates</td>
<td>$326.76</td>
<td>$296.26</td>
</tr>
</tbody>
</table>

Table 18: Comparison of Expected DOT Costs
In Table 18, we present the expected (ex-post) DOT cost in each case. This is the expectation of the amount that the DOT would pay the winning bidder $q_a^c h_c^w + q_r^a h_r^w$ at the equilibrium bidding strategy in each setting, taken with respect to the distribution of the type of the lowest (winning) bidder. When bidders are risk neutral ($\gamma = 0$), the equilibrium cost to the DOT does not change when the DOT improves its quantity estimates. The reason for this is that since $\gamma = 0$, the variance term in Equation (55) is zero regardless of the level of the noise in quantity predictions. As the bidders’ quantity expectations $E[q_a^c]$ and $E[q_r^a]$ are unchanged, the expected revenue of the winning bidder (corresponding to the expected cost to the DOT) is unchanged as well.

![Equilibrium DOT Cost/Bidder Revenue by Bidder Type](image)

(a) Risk Neutral Bidders  
(b) Risk Averse Bidders: $\gamma = 0.05$

Figure 12: Equilibrium DOT Cost/Bidder Revenue by Bidder Type

![Equilibrium Score Functions by Bidder Type](image)

(a) Risk Neutral Bidders  
(b) Risk Averse Bidders: $\gamma = 0.05$

Figure 13: Equilibrium Score Functions by Bidder Type

In Figure 12a, we plot the revenue that each type of bidder expects to get in equilibrium when bidders are risk neutral. The red line corresponds to the baseline setting, in which the DOT underestimates the ex-post quantity of concrete, and overestimates the ex-post quantity of traffic cones. The black line corresponds to the counterfactual in which both quantities are precisely estimated.

\footnote{In order to simulate equilibria, we need to assume a distribution of bidder types. For this example, we assume that bidder types are distributed according to a truncated lognormal distribution, $\alpha \sim \text{LogNormal}(0, 0.2)$ that is bounded from above by 2.5. There is nothing special about this particular choice, and we could easily have made others with similar results.}
estimated, and bidders have no residual uncertainty about what the quantities will be. While the 
ex-post cost to the DOT is the same whether or not the DOT quantity estimates are correct, the unit 
bids and resulting scores that bidders submit are different. In Figure 13a, we plot the equilibrium 
score for each bidder type when bidders are risk neutral. The score at every bidder type is smaller 
under the baseline than under the counterfactual in which the DOT discerns ex-post quantities. 
This is because the scores in the counterfactual correspond to the bidders’ expected revenues, while 
the scores in the baseline multiply bids that are skewed to up-weight over-running items by their 
under-estimated DOT quantities. See the appendix for a full derivation and discussion of the risk 
neutral case.

![Figure 14a](image1.png)  
![Figure 14b](image2.png)  

Figure 14: Equilibrium Unit Bids by Bidder Type

Figure 14a plots the unit bid that each type of bidder submits in equilibrium when bidders are 
risk neutral. As before, the red lines correspond to the baseline setting in which the DOT mis-
estimates quantities, whereas the black lines correspond to the counterfactual setting in which the 
DOT discerns ex-post quantities perfectly. The solid line in each case corresponds to the unit bid 
for concrete $b_c(\alpha)$ that each $\alpha$ type of bidder submits in equilibrium. The dashed line corresponds 
to the equilibrium unit bid for traffic cones $b_r(\alpha)$ for each bidder type. Notably, in every case, 
the optimal bid for each bidder puts the maximum possible amount (conditional on the bidder’s 
equilibrium score) on the item that is predicted to over-run the most, and $0$ on the other item. 
This is a direct implication of optimal bidding by risk neutral bidders, absent an external impetus 
to do otherwise. As noted by Athey and Levin (2001), this suggests that the observations of interior 
or intermediately-skewed bids in our data, as well as in Athey and Levin’s, are inconsistent with 
a model of risk neutral bidders. Other work, such as Bajari, Houghton, and Tadelis (2014) have 
rationalized interior bids by modeling a heuristic penalty for extreme skewing that represents a fear 
of regulatory rebuke. However, no significant regulatory enforcement against bid skewing has ever 
been exercised by MassDOT, and discussions of bidding incentives in related papers as well as in 
Athey and Levin (2001) suggest that risk avoidance is a more likely dominant motive.

In Figures 12b, 13b and 14b, we plot the equilibrium revenue, score and bid for every bidder 
type, when bidders are risk averse with the CARA coefficient $\gamma = 0.05$. Unlike the risk-neutral
case, the DOT’s elimination of uncertainty regarding quantities has a tangible impact on DOT costs. When the DOT eliminates quantity risk for the bidders, it substantially increases the value of the project for all of the bidders, causing more competitive bidding behavior. Seen another way, uncertainty regarding ex-post quantities imposes a cost to the bidders, on top of the cost of implementing the project upon winning. In equilibrium, bidders submit bids that allow them to recover all of their costs (plus a mark-up). When uncertainty is eliminated, the cost of the project decreases, and so the total revenue needed to recover each bidder’s costs decreases as well. Note, also, that the elimination of uncertainty results in different levels of skewing across the unit bids of different items. Whereas under the baseline, bidders with types \( \alpha > 1.6 \) place increasing interior bids on traffic cones, when risk is eliminated, this is no longer the case. However, this is subject to a tie breaking rule – when the DOT perfectly predicts ex-post quantities, there are no over-runs, and so there is no meaningful different to overbid on one item over the other. The analysis of the optimal bid (conditional on a score) here is analogous to that under risk neutrality, and so we defer details to the appendix.

<table>
<thead>
<tr>
<th>CARA Coeff</th>
<th>Baseline</th>
<th>No Quantity Risk</th>
<th>Pct Diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$326.76</td>
<td>$326.76</td>
<td>0%</td>
</tr>
<tr>
<td>0.001</td>
<td>$326.04</td>
<td>$325.62</td>
<td>0.13%</td>
</tr>
<tr>
<td>0.005</td>
<td>$323.49</td>
<td>$321.41</td>
<td>0.64%</td>
</tr>
<tr>
<td>0.01</td>
<td>$321.01</td>
<td>$316.88</td>
<td>1.29%</td>
</tr>
<tr>
<td><strong>0.05</strong></td>
<td><strong>$317.32</strong></td>
<td><strong>$296.26</strong></td>
<td><strong>6.64%</strong></td>
</tr>
<tr>
<td>0.10</td>
<td>$319.83</td>
<td>$285.57</td>
<td>10.71%</td>
</tr>
</tbody>
</table>

Table 19: Comparison of expected DOT costs under different levels of bidder risk aversion

While the general observation that reducing uncertainty may result in meaningful cost savings to the DOT, the degree of these savings depends on the baseline level of uncertainty in each project, as well as the degree of bidders’ risk aversion and the level of competition in each auction (constituted by the distribution of cost types and the number of participating bidders). To illustrate this, we repeat the exercise summarized in Table 18 over different degrees of risk aversion and different levels of uncertainty. In Table 19, we present the expected DOT cost under the baseline example and under the counterfactual in which the DOT eliminates quantity risk, as well as the percent difference between the two, for a range of CARA coefficients.\(^{39}\) The bolded row with a CARA coefficient of 0.05 corresponds to the right hand column of Table 18.

\(^{39}\)That is, in the baseline, the DOT posts quantity estimates \( q^e_c = 10 \) and \( q^e_r = 20 \), while bidders predict that \( E[q^a_c] = 12 \) and \( E[q^a_r] = 18 \) with \( \sigma^2_c = 2 \) and \( \sigma^2_r = 1 \). In the No Quantity Risk counterfactual, the DOT discerns that \( q^e_c = q^a_c = 12 \) and \( q^e_r = q^a_r = 18 \), so that \( \sigma^2_c = \sigma^2_r = 0 \).
A.4 Worked Out Example of Risk Neutral Bidding

Two risk-neutral bidders compete for a project that requires two types of inputs to complete: concrete and traffic cones. The DOT estimates that 10 tons of concrete and 20 traffic cones will be necessary to complete the project. However, the bidders (both) anticipate that the actual quantities that will be used – random variables that we will denote $q_a^c$ and $q_a^r$ for concrete and traffic cones, respectively – are distributed with means $\mathbb{E}[q_a^c] = 12$ and $\mathbb{E}[q_a^r] = 10$. We will assume that the actual quantities are exogenous to the bidding process, and do not depend on who wins the auction in any way.

The bidders differ in their private costs for the materials (including overhead, etc.): each bidder $i$ incurs a privately known flat unit cost $c^i_c$ for each unit of concrete and $c^i_r$ for each traffic cone used. Thus, at the time of bidding, each bidder $i$ expects to incur a total cost

$$\theta^i \equiv \mathbb{E} \left[ q_a^c c^i_c + q_a^r c^i_r \right] = 12c^i_c + 10c^i_r,$$

if she were to win the auction. Each bidder $i$ submits a unit bid for each of the items: $b^i_c$ and $b^i_r$. The winner of the auction is then chosen on the basis of her score: the sum of her unit bids multiplied the DOT’s quantity estimates:

$$s^i = 10b^i_c + 20b^i_r.$$

Once a winner is selected, she will implement the project and earn net profits of her unit bids, less the unit costs of each item, multiplied by the realized quantities of each item that are ultimately used. At the time of bidding, these quantities are unrealized samples of random variables. However, as the bidders are risk-neutral, they consider the expected value of profits to make their bidding decisions:

$$E[\pi(b^i_c, b^i_r)|c^i_c, c^i_r] = \mathbb{E} \left[ (q_a^c b^i_c + q_a^r b^i_r) - (q_a^c c^i_c + q_a^r c^i_r) \right] \times \text{Prob}(s^i < s^j)$$

The key intuition for bid skewing is as follows. Suppose that the bidders’ expectations of the actual quantities to be used are accurate. Then for any score $s$ that bidder $i$ deems competitive, she can construct unit bids that maximize her ex-post profits if she wins the auction. For example, suppose that bidder $i$ has unit costs $c^i_c = $70 and $c^i_r = $3, and she has decided to submit a score of $1000. She could bid her costs with a $5 markup on concrete and a $9.50 markup on traffic cones: $b^i_c = $75 and $b^i_r = $12.50, yielding a net profit of $155. However, if instead, she bids $b^i_c = $99.98 and $b^i_r = $0.01, bidder $i$ could submit the same score, but earn a profit of nearly $330 if she wins.

This logic suggests that the DOT’s inaccurate estimates of item quantities enable bidders to extract surplus profits without ceding a competitive edge. If the DOT were able to predict the actual
quantities correctly, it would eliminate the possibility of bid skewing. In order for bidder $i$ to submit a score of $1000$ in this case, she would need to choose unit bids such that $12b_c^i + 20b_r^i = 1000$—the exact revenue that she would earn upon winning the auction. She could still bid $b_r^i = 0.01$, for example, but then she would need to bid $b_c^i = 83.33$, resulting in a revenue of $1000$ and a profit of $130$ if she wins the auction. A quick inspection shows that no choice of $b_c^i$ and $b_r^i$ could improve her expected revenue at the same score.

It would follow that when bidders have more accurate assessments of what the actual item quantities will be— as is generally considered to be the case— bids with apparent skewing are materially more costly to the DOT. If the bidders were to share their expectations truthfully with the DOT, it appears that a lower total cost might be incurred without affecting the level of competition.

However, this intuition does not take into account the equilibrium effect that a change in DOT quantity estimates would have on the competitive choice of score. It is not true that if a score of $1000$ is optimal for bidder $i$ under inaccurate DOT quantity estimates, then it will remain optimal under accurate DOT estimates as well. As we demonstrate below, when equilibrium score selection is taken into consideration, the apparent possibility of extracting higher revenues by skewing unit bids is shut down entirely.

To illustrate this point, we derive the equilibrium bidding strategy for each bidder in our example. In order to close the model, we need to make an assumption about the bidders’ beliefs over their opponents’ costs. Bidder $i$’s expected total cost for the project $\theta^i$ is fixed at the time of bidding, and does not depend on her unit bids. For simplicity, we will assume that these expected total costs are distributed according to some commonly known distribution: $\theta \sim F[\theta, \bar{\theta}]$.

By application of Asker and Cantillon (2010), there is a unique (up to payoff equivalence) monotonic equilibrium in which each bidder of type $\theta$ submits a unique equilibrium score $s(\theta)$, using unit bids that maximize her expected profits conditional on winning, and add up to $s(\theta)$. That is, in equilibrium, each bidder $i$ submits a vector of bids $\{b_c(\theta^i), b_r(\theta^i)\}$ such that:

$$\{b_c(\theta^i), b_r(\theta^i)\} = \arg \max_{\{b_c, b_r\}} \left\{12b_c + 40b_r - \theta^i \right\} \text{ s.t. } 10b_c + 50b_r = s(\theta^i).$$

Solving this, we quickly see that at the optimum, $b_r(\theta^i) = 0$ and $b_c(\theta^i) = s(\theta^i)/10$ (to see this, note that if $b_r = 0$, then the bidder earns a revenue of $\frac{12}{10} \cdot s(\theta^i)$ whereas if $b_c = 0$, then the bidder earns a revenue of $\frac{40}{50} \cdot s(\theta^i)$.)

The equilibrium can therefore be characterized by the optimality of $s(\theta)$ with respect to the expected profits of a bidder with expected total cost $\theta$:

$$E[\pi(s(\theta^i))|\theta^i] = \left(\frac{12}{10} \cdot s(\theta^i) - \theta^i\right) \cdot \text{Prob } (s(\theta^i) < s(\theta^i))$$

$$= \left(\frac{12}{10} \cdot s(\theta^i) - \theta^i\right) \cdot (1 - F(\theta^i)), \quad (56)$$

$$= \left(\frac{12}{10} \cdot s(\theta^i) - \theta^i\right) \cdot (1 - F(\theta^i)), \quad (57)$$

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where the second equality follows from the strict monotonicity of the equilibrium.\footnote{More concretely, a monotonic equilibrium requires that for any \( \theta' > \theta \), \( s(\theta') > s(\theta) \). Therefore, the probability that \( s(\theta') \) is lower than \( s(\theta) \) is equal to the probability that \( \theta' \) is lower than \( \theta \).}

As in a standard first price auction, the optimality of the score mapping is characterized by the first order condition of expected profits with respect to \( s(\theta) \):

\[
\frac{\partial E[\pi(\tilde{s}, \theta)]}{\partial \tilde{s}} |_{\tilde{s} = s(\theta)} = 0.
\]

Solving the resulting differential equation, we obtain:

\[
s(\theta) = \frac{10}{12} \left[ \theta + \int_{\theta}^{\tilde{\theta}} \frac{1 - F(\tilde{\theta})}{1 - F(\theta)} \, d\tilde{\theta} \right].
\]

Thus, each bidder \( i \) will bid \( b_c(\theta^i) = \frac{s(\theta^i)}{10} \) and \( b_r(\theta) = 0 \). If bidder \( i \) wins the auction, she expects to earn a markup of:

\[
E[\pi(\theta^i)] = 12 \cdot \frac{s(\theta^i)}{10} - \theta^i
\]

More generally, no matter what the quantities projected by the DOT are – entirely correct or wildly inaccurate – the winner of the auction and the markup that she will earn in equilibrium will be the same.

In particular, writing \( q^e_c \) and \( q^e_r \) for the DOT’s quantity projections (so that a bidder’s score is given by \( s = b_c q^e_c + b_r q^e_r \)) and \( q^b_c \) and \( q^b_r \) for the bidders’ expectations for the actual quantities, the equilibrium score function can be written:

\[
s(\theta) = \min \left\{ \frac{q^e_c}{q^b_c}, \frac{q^e_r}{q^b_r} \right\} \left[ \theta + \int_{\theta}^{\tilde{\theta}} \frac{1 - F(\tilde{\theta})}{1 - F(\theta)} \, d\tilde{\theta} \right].
\]

Suppose that \( \frac{q^e_c}{q^b_c} \leq \frac{q^e_r}{q^b_r} \). Then bidder \( i \) will bid \( b^*_c(\theta^i) = \frac{s(\theta^i)}{q^b_c} \) and \( b^*_r(\theta^i) = 0 \). Consequently, if bidder \( i \) wins, she will be paid \( q^b_c \cdot b^*_c(\theta^i) = \left[ \theta^i + \int_{\theta^i}^{\tilde{\theta}} \frac{1 - F(\tilde{\theta})}{1 - F(\theta^i)} \, d\tilde{\theta} \right] \) as in our example.

The probability of winning is determined by the probability of having the lowest cost type, in equilibrium, and so this too is unaffected by the DOT’s quantity estimates. That is, the level of competition and the degree of markups extracted by the bidders is determined entirely by the density of the distribution of expected total costs among the competitors. The more likely it is that bidders have similar costs, the lower the markups that the bidders can extract. However, regardless of whether the DOT posts accurate quantity estimates—in which case, bidders cannot
benefit from skewing their unit bids at any score—or not, the expected cost of the project to the DOT will be the same in equilibrium. Therefore, a mathematically unbalanced bid, while indicative of a discrepancy in the quantity estimates made by the bidders and the DOT, is not indicative of a material loss to the government.

A.5 Discussion of Policy Inefficacy If Bidders are Risk Neutral

In the body of our paper, we present three counterfactual policy proposals: (1) reducing the latent uncertainty about item quantities; (2) switching to a lump sum auction; (3) subsidizing entry costs in order to incentivize additional entry. We show that when bidders are risk averse (and in particular, under the estimated level of risk aversion), these policies each have a significant impact on DOT spending in equilibrium. In this section, we argue formally that risk aversion is key to these results. In particular, if bidders are instead risk neutral, then the effect of all of these policies is unambiguously null in equilibrium.

The key intuition to these results is as follows. The equilibrium construction in Appendix A.4 would be almost identical with \( T \) items, rather than 2. For risk neutral bidders, the choice of bid vector that maximizes the “inner” optimization problem conditional on a score is independent of the bidder’s type: at the optimum, each bidder bids her entire score (normalized by the DOT engineer’s quantity projection) on the item that will over-run the most in expectation, and zero on every other item.

Thus, bidding is effectively one-dimensional, and all of the properties of standard single-item first price auctions with risk neutral bidders apply. In particular, not only would the policies to reduce uncertainty or switch to a lump sum auction be ineffective (which follows directly from the model as “risk” does not enter into bidder preferences), but so would a policy to subsidize bidders. This result is a consequence of revenue equivalence: the decrease in expected costs from the entry of an additional bidder is equal to the expected profit that this bidder would earn upon entering.\(^{41}\) As such, the equilibrium number of entrants to a given auction is necessarily efficient: incentivizing an additional bidder would cost the full additional surplus that this bidder would bring. By contrast, revenue equivalence does not apply with risk averse bidders, and the additional bidder’s expected utility from entering may be lower than the decrease in expected costs if she enters.

B Views of Bid Skewing by Contractors and MassDOT Managers

Bid Skewing Among Contractors

The practice of unbalanced bidding—or bid skewing—in scaling auctions appears, in the words of one review, “to be ubiquitous” (Skitmore and Cattell (2013)). References to bid skewing in

\(^{41}\)See Klemperer (1999) for a fuller but still intuitive discussion of this.
operations research and construction management journals date as far back as 1935 and as recently as 2010. A key component of skewing is the bidders’ ability to predict quantity over/under-runs and optimize accordingly. Stark (1974), for instance, characterizes contemporary accounts of bidding:

> Knowledgeable contractors independently assess quantities searching for items apt to seriously under-run. By setting modest unit bids for these items they can considerably enhance the competitiveness of their total bid.

Uncertainty regarding the quantities that will ultimately be used presents a challenge to optimal bid-skewing, however. In an overview of “modern” highway construction planning, Tait (1971) writes:

> ...there is a risk in manipulating rates independently of true cost, for the quantities schedule in the bill of quantities are only estimates and significant differences may be found in the actual quantities measured in the works and on which payment would be based.

In order to manage the complexities of bid selection, contractors often employ experts and software geared for statistical prediction and optimization. Discussing the use of his algorithm for optimal bidding in consulting for a large construction firm, Stark (1974) notes a manager’s prediction that such software would soon become widespread—reducing asymmetries between bidders and increasing allocative efficiency in the industry.

> ...since the model was public and others might find it useful as well, it had the longer term promise of eroding some uncertainties and irrelevancies in the tendering process. Their elimination...increased the likelihood that fewer contracts would be awarded by chance and that his firm would be a beneficiary.

Since then, an assortment of decision support tools for estimating item quantities and optimizing bids has become widely available. A search on Capterra, a web platform that facilitates research for business software buyers, yields 181 distinct results. In a survey on construction management software trends, Capterra estimates that contractors spend an average $2,700 annually on software. The top 3 platforms command a market share of 36% and surveyed firms report having used their current software for about 2 years—suggesting a competitive environment. Asked what was most improved by the software, a leading 21% of respondents said, “estimating accuracy”, while 14% (in third place) said “bidding”.

MassDOT Challenges to Bid Skewing

Concerns that sophisticated bidding strategies may allow contractors to extract excessively large payments have led to a number of lawsuits about MassDOT’s right to reject suspicious bids. The Federal Highway Administration (FHWA) has explicit policies that allow officials to reject bids
that are deemed manipulative. However, the legal burden of proof for a manipulative bid is quite high. In order for a bid to be legally rejected, it must be proven to be *materially unbalanced*.42

A bid is materially unbalanced if there is a reasonable doubt that award to the bidder ... will result in the lowest ultimate cost to the Government. Consequently, a materially unbalanced bid may not be accepted.13

However, as the definition for material unbalancedness is very broad, FHWA statute requires that a bid be *mathematically* unbalanced as a precondition. A *mathematically unbalanced* bid is defined as one, “structured on the basis of nominal prices for some work and inflated prices for other work.”44 In other words, it is a bid that appears to be strategically skewed. In order to discourage bid skewing, many regional DOTs use concrete criteria to define mathematically unbalanced bids. In Massachusetts, a bid is considered mathematically unbalanced if it contains any line-item for which the unit bid is (1) over (under) the office cost estimate and (2) over (under) the average unit bid of bidders ranked 2-5 by more than 25%.

In principle, a mathematically unbalanced bid elicits a flag for MassDOT officials to examine the possibility of material unbalancedness. However, in practice, such bids are ubiquitous, and substantial challenges by MassDOT are very rare. In our data, only about 20% of projects do not have a single item breaking MassDOT’s overbidding rule, and only about 10% of projects do not have a single item breaking the underbidding rule. Indeed, most projects have a substantial portion of unit bids that should trigger a mathematical unbalancedness flag. However, only 2.5% of projects have seen bidders rejected across all justifications, a handful of which were due to unbalanced bids.45

**The Difficulty of Determining ‘Materially Unbalanced’ Bids**

A primary reason that so few mathematically unbalanced bids are penalized is that material unbalancedness is very hard to prove. In a precedent-setting 1984 case, the Boston Water and Sewer Commission was sued by the second-lowest bidder for awarding a contract to R.J. Longo Construction Co., Inc., a contractor who had the lowest total bid along with a penny bid. The Massachusetts Superior Court ruled that the Commission acted correctly, since the Commission saw no evidence that the penny bid would generate losses for the state. More specifically, no convincing evidence was presented that if the penny bid did generate losses, the losses would exceed the premium on construction that the second-lowest bidder wanted to charge (Mass Superior Court, 1984).46 In

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42See Federal Acquisition Regulations, Sec. 14.201-6(e)(2) for sealed bids in general and Sec. 36.205(d) for construction specifically (Cohen Seglias Pallas Greenhall and Furman PC (2018)).
45MassDOT does not reject individual bidders, but rather withdraws the project from auction and possibly resubmits it for auction after a revision of the project spec.
46In response to this case, MassDOT inserted the following clause into Subsection 4.06 of the MassDOT Standard Specifications for Highways and Bridges: “No adjustment will be made for any item of work identified as having an unrealistic unit price as described in Subsection 4.04.” This clause, inserted in the Supplemental Specifications dated December 11, 2002, made it difficult for contractors to renegotiate the
January 2017, MassDOT attempted to require a minimum bid for every unit price item in a various locations contract due to bid skewing concerns. SPS New England, Inc. protested, arguing that such rules preclude the project from being awarded to the lowest responsible bidder. The Massachusetts Assistant Attorney General ruled in favor of the contractor on August 1, 2017.

In fact, as we show in Appendix A.2, there is a theoretical basis to question the relationship between mathematical and material unbalancedness. As we demonstrate, bid skewing plays dual roles in bidders’ strategic behavior. On the one hand, bidders extract higher ex-post profits by placing higher bids on items that they predict will over-run in quantity. On the other hand, bidders reduce ex-ante risk by placing lower bids on items, regarding which they are particularly uncertain. Moreover, when bidders are similarly informed regarding ex-post quantities, the profits from predicting over-runs are largely competed away in equilibrium, but the reduction in ex-ante risk is passed on to MassDOT in the form of cost-savings.

C Solving Portfolio Problems

We present a fast, deterministic algorithm to solve the constrained quadratic programs found in the bidders’ portfolio problems. Given the independence of items within a project, each problem can be represented in the form:

$$\max_{\{x_i\}} \sum_i a_i x_i - b_i x_i^2 \quad \text{subject to} \quad \begin{cases} \sum_i q_i x_i = s \\ x_i \geq 0 \text{ for each } i \end{cases}$$

The primal formulation of this problem is

$$\max \min_{\{x_i\}} \left\{ \sum_i f_i(x) + v \left( q x - s \right) \right\}$$

where

$$f_i(x) = a_i x - b_i x^2 + p_i(x)$$

and

$$p_i(x) = \begin{cases} \infty & \text{if } x_i < 0 \\ 0 & \text{otherwise} \end{cases}$$

where $v$ is the Lagrange multiplier on the linear constraint. By well known results, we can instead unit price of penny bid items during the course of construction. An internal MassDOT memo from the time shows that Construction Industries of Massachusetts (CIM) requested that this clause be removed. One MassDOT engineer disagreed, writing that “if it is determined that MHD should modify Subsection 4.06 as requested by CIM it should be noted that the Department may not necessarily be awarding the contract to the lowest responsible bidder as required.” The clause was removed from Subsection 4.06 in the June 15, 2012 Supplemental Specifications.
solve the dual problem:

\[
\min_v \max_{\{x_i\}} \left\{ \sum_i f_i(x) + v \left( q^T x - s \right) \right\}
\]

The First Order Conditions of \( g(x) \) are given by \( \frac{\partial g(x)}{\partial x_i} = a_i - 2bx_i + vq_i = 0 \) and so, at the optimum:

\[
x_i^* = \max \left\{ \frac{a_i + vq_i}{2b_i}, 0 \right\}.
\]

Substituting \( x^* \) into the dual objective, we obtain:

\[
\min_v \left\{ \sum_i h_i(x_i^*) + v(s) \right\}
\]

where: \( h_i(x_i^*) = \begin{cases} \frac{(a_i + vq_i)^2}{2b_i} - b_i \left( \frac{a_i + vq_i}{2b_i} \right)^2 & \text{if } \frac{a_i + vq_i}{2b_i} > 0 \\ 0 & \text{otherwise.} \end{cases} \)

Simplifying this further,

\[
h_i(x_i^*) = \begin{cases} \frac{1}{4b_i} (a_i + vq_i)^2 & \text{if } \frac{a_i + vq_i}{2b_i} > 0 \\ 0 & \text{otherwise.} \end{cases}
\]

Thus, the solution to the original problem is the \( v_k^* \) that minimizes Equation (*) with \( k \) non-zero components with the form \( x_i^* = \frac{a_i + v_k^* q_i}{2b_i} \). Noting that \( \frac{a_i + vq_i}{2b_i} > 0 \iff v > -\frac{a_i}{q_i} \), we propose the following algorithm for solving the problem:

1. Rank \( \{i\} \) in order of \( -\frac{a_i}{q_i} \) (lowest to highest). Note that under this sorting, for any \( \tilde{v} \), if \( \tilde{v} \leq -\frac{a_j}{q_j} \) for some \( j \), then \( \tilde{v} \leq -\frac{a_k}{q_k} \) for all \( k > j \).

2. For each \( k \), let \( \tilde{v}_k \) to be the value of \( v \) that minimizes (*) on the interval \( (-\frac{a_k}{q_k}, -\frac{a_{k+1}}{q_{k+1}}) \). Do a line search over \( k \) to find the minimizer.

Note that for any \( k \) in Step 2., there is a closed form solution to \( \tilde{v}_k \):

\[
\tilde{v}_k = \arg \min_v \left\{ \sum_{i \leq k} \frac{1}{4b_i} (a_i + \tilde{v} q_i)^2 - \tilde{v} k s \right\}.
\]

This is a sum of quadratics (e.g. a quadratic), and so we find the optimum by taking the FOC:

\[
\tilde{v}_k^* = \min \left\{ \frac{2s - \sum_{i \leq k} \frac{a_i}{q_i}}{\sum_{i \leq k} \frac{a_i}{q_i}}, -\frac{a_{k+1}}{q_{k+1}} \right\}.
\]
**Edge Cases**  This algorithm above will work so long as \(\frac{a_i + vq_i}{2b_i}\) is well defined – that is so long as \(b_i > 0\). When there is (at least one) element \(i\) such that \(b_i = 0\) (and so it is linear), the optimal solution will stop propagating \(v_k\)'s as soon as it hits the first linear element in the \(-a_i/q_i\) rank order. At that point (say the linear element is the \(k\)th one): \(v_k = -a_k/q_k\) and \(x_k = \frac{s-\sum_{i\leq k} q_i x_i^*}{q_k}\).

**Adding Item-Level Constraints**  Suppose that we add item-specific constraints, so that our problem is:

\[
\max_{\{x_i\}} \sum_i a_i x_i - b_i x_i^2; \text{ subject to } \begin{cases} 
\sum_i q_i x_i = s \\
x_i \geq r_i \text{ for each } i
\end{cases}
\]

where \(r_i > 0\) is some (known) number for each component \(i\).

To use our algorithm above, we simply transform \(x\) into a new variable: \(y = x - r\)

\[
\max_{\{y_i\}} \sum_i a_i (y_i + r_i) - b_i (y_i + r_i)^2; \text{ subject to } \begin{cases} 
\sum_i q_i (y_i + r_i) = s \\
y_i \geq 0 \text{ for each } i
\end{cases}
\]

Simplifying, we see that this fits right into our previous framework:

\[
\max_{\{y_i\}} \sum_i \tilde{a}_i y_i - \tilde{b}_i y_i^2 + \tilde{C}_i; \text{ subject to } \begin{cases} 
\sum_i q_i y_i = \tilde{s} \\
y_i \geq 0 \text{ for each } i
\end{cases}
\]

where:

\[
\begin{aligned}
\tilde{a}_i &= a_i - 2b_i r_i \\
\tilde{b}_i &= b_i \\
\tilde{C}_i &= a_i r_i - b_i r_i^2 \\
\tilde{s} &= s - \sum_i q_i r_i
\end{aligned}
\]

Note that \(\tilde{C}\) is a constant and so does not affect optimization.