Robust Bounds for Welfare Analysis

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Motivation

- Many papers in economics have the following structure:
  1. A policy (*e.g.*, tax/subsidy) was implemented.
  2. Using prices and quantities before and after, estimate demand.
  3. Impute the change in welfare + compare to costs/revenues.

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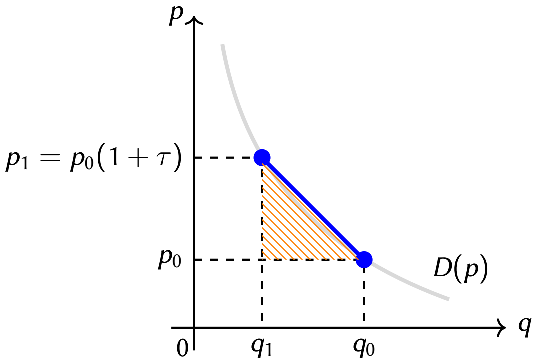
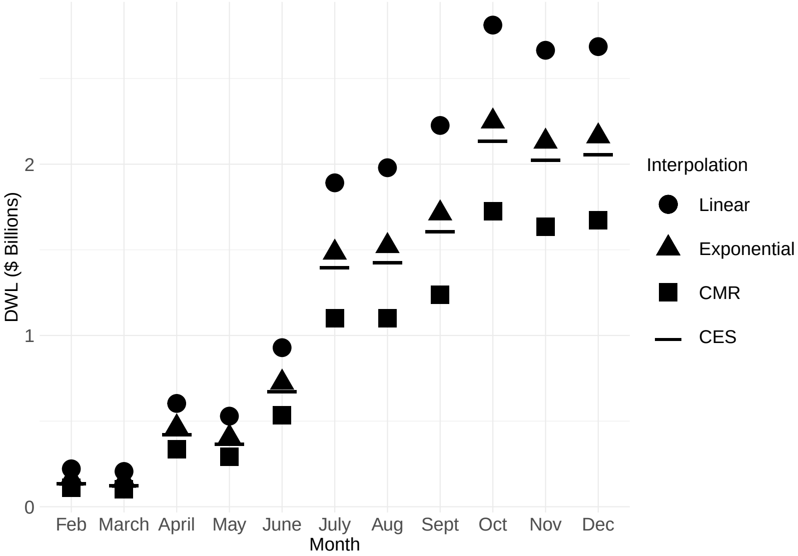
  $\rightarrow$ Functional forms (*e.g.*, CES or linear demand) are often assumed for convenience.

Example: evaluating the deadweight loss of the Trump tariffs



- Amiti, Redding and Weinstein (2019)
- **Setting:** 2018 trade war involved tariffs as high as 30–50%.
- **Question:** What was the DWL?
- **Approach:** Compare monthly prices & quantities by item in 2017 vs. 2018.
- **Method:** Approximate $D(p)$ with a linear curve; integrate under the curve.

Bounding the DWL across countries and products



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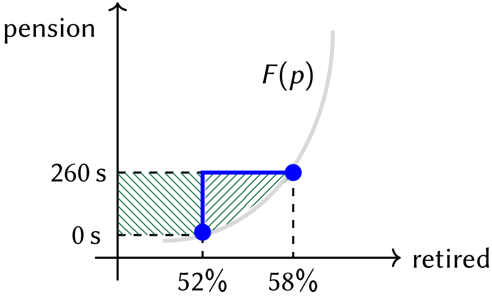
  $\rightarrow$ Conservative bounds in lieu of assumptions are often extreme.

Example: WTP of 1911 UK pension recipients



- Giesecke and Jäger (2021)
- **Setting:** Pensions created for poor 70+ year olds in 1911.
- **Question:** What is the MVPF of the pension policy?
- **Approach:** MVPF = (WTP for not working) / (cost of pension).
- **Method:** Compute % marginal workers via RD; assume marginal workers' WTP = 0.

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- Measuring welfare requires taking a stance on what the demand curve looks like at unobserved points.
    - Functional forms (*e.g.*, CES or linear demand) are often assumed for convenience.
    - Conservative bounds in lieu of assumptions are often extreme.

  $\rightsquigarrow$ Is there a more principled way to engage with assumptions and evaluate welfare?

This paper

- Instead of interpolating to get a welfare estimate, we establish **welfare bounds**.
  - These bounds are **robust**: they give the *best-case* and *worst-case* welfare estimates that are consistent with a set of pre-specified economic assumptions.
  - These bounds are also **simple**: we can compute them in closed form.

"Economists have made remarkable progress over the last several decades in developing empirical techniques that provide compelling **evidence of causal effects**—the so-called **"credibility revolution"** in empirical work…

But while it is interesting and important to know what the effects of a policy are, we are often also interested in a **normative question** as well: Is the policy a **good** idea or a **bad** idea?

…What is the **welfare impact of the policy**?"

—Finkelstein and Hendren (2020)

This is a tool for empirical microeconomists

- Our bounds apply directly to settings with:
  (i) exogenous policy shocks/experiments/quasi-experiments;
  (ii) measurements of “price” and “quantity,” before and after the policy shock; and
  (iii) interest in effects on consumer surplus (or other welfare measures).

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- We show how our bounds can be applied to a variety of settings across literatures:
  #1. deadweight loss of import tariffs (Amiti, Redding and Weinstein, 2019)
  #2. welfare impact of energy subsidies (Hahn and Metcalfe, 2021)
  #3. willingness to pay for the Old-Age Pension Act (Giesecke and Jäger, 2021)
  #4. marginal excess burden of income taxation (Feldstein, 1999)

This is an application of information design for econometrics

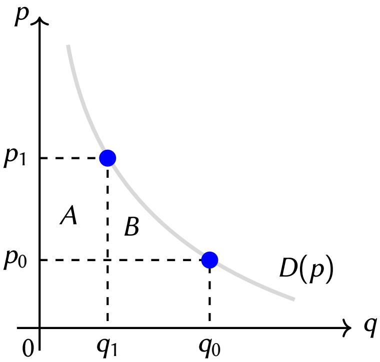
- Applies ideas from information design to interpret econometrics:
  - **Key idea:** maximize/minimize welfare over the space of *feasible* demand curves.
  - **Main result:** the max/min bounds on welfare are attained by simple one-piece and two-piece interpolations for a number of (arguably) useful restrictions on demand.

- Applies ideas from information design to interpret econometrics:
  - **Key idea:** maximize/minimize welfare over the space of *feasible* demand curves.
  - **Main result:** the max/min bounds on welfare are attained by simple one-piece and two-piece interpolations for a number of (arguably) useful restrictions on demand.
  - **Bonus:** our bounds shed light on the implications of commonly used demand curves.
    - $\rightsquigarrow$ *E.g.*, CES interpolation yields the *smallest* welfare estimate among all possible interpolations, assuming that the demand curve satisfies Marshall's second law.

Basic model

An analyst observes 2 points on a demand curve: (p_0, q_0) and (p_1, q_1).

Question. What is the change in consumer surplus from (p_0, q_0) to (p_1, q_1)?

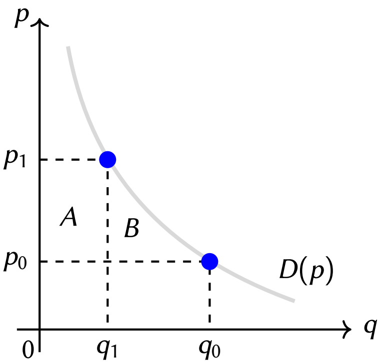


- **Main challenge:** $D(p)$ isn't observed.
- With $D(p)$, change in CS is equal to

$$\underbrace{\text{area } A}_{=(p_1-p_0)q_1} + \text{area } B = \int_{p_0}^{p_1} D(p)\,\mathrm{d}p.$$

- Equivalently, we want to *bound* area B.

- Using only the fact that the demand curve is decreasing, the analyst can establish bounds on the change in welfare (Fogel, 1964; Varian, 1985).



Bounds without additional assumptions

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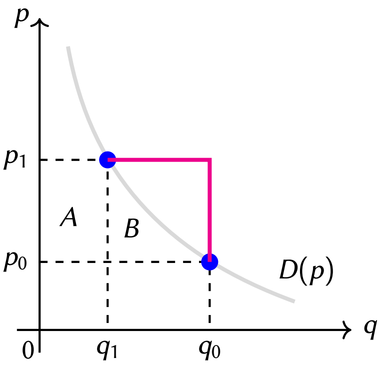

- An upper bound on area B is

$$\text{area } B \leq (p_1 - p_0) \times (q_0 - q_1)\,.$$

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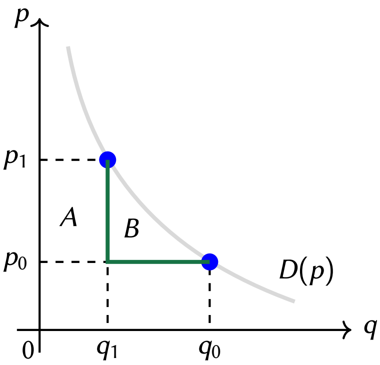

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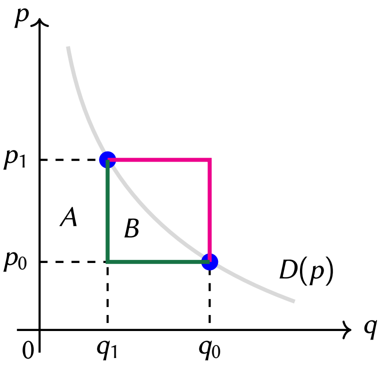

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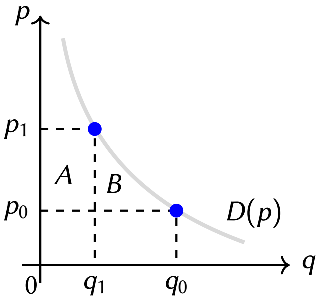
- These bounds are attained only when **elasticities are equal to 0 or $-\infty$**.

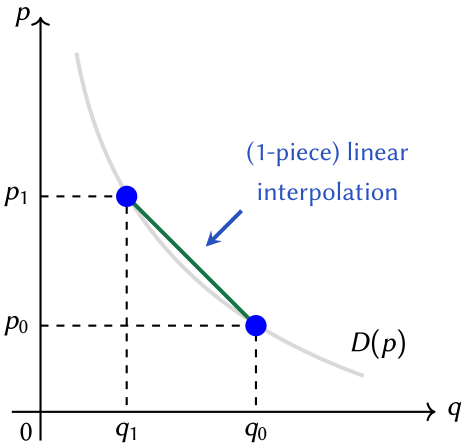
An analyst observes 2 points on a demand curve: (p_0, q_0) and (p_1, q_1).

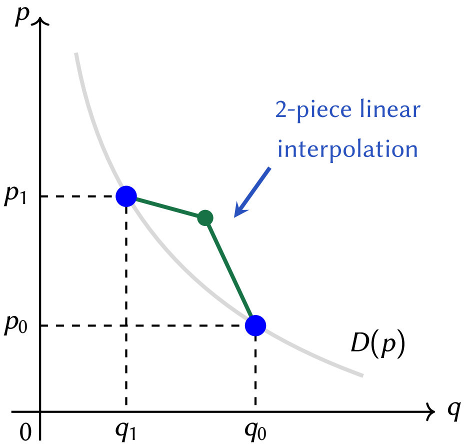
We assume that elasticities between (p_0, q_0) and (p_1, q_1) lie in the interval $[\underline{\varepsilon}, \overline{\varepsilon}] \subset \mathbb{R}_{\leq 0}$.

Question. What is the change in consumer surplus from (p_0, q_0) to (p_1, q_1)?



Defining 1-piece and 2-piece interpolations

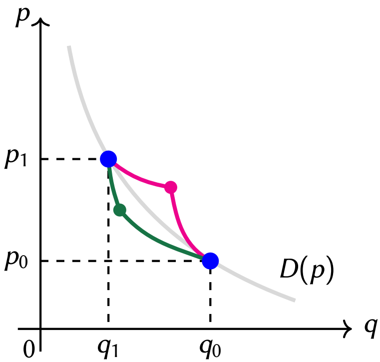




Theorem 1 (welfare bounds).

The upper and lower bounds for the change in consumer surplus are attained by **2-piece CES interpolations**.

Skip proof



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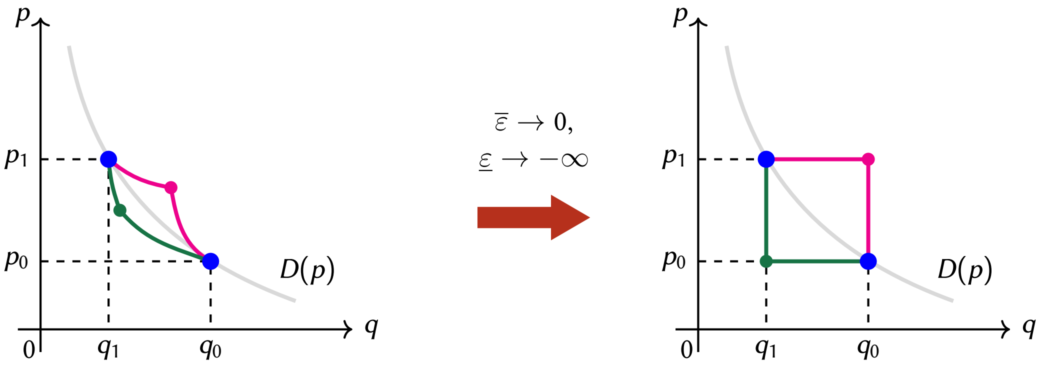
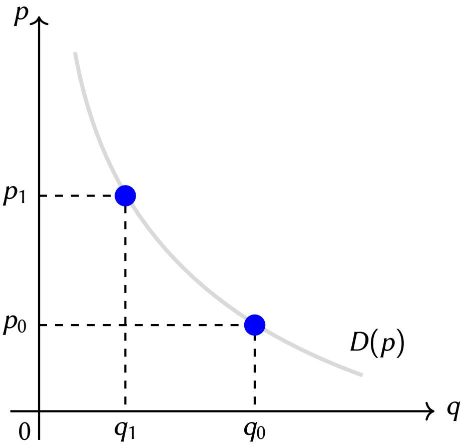
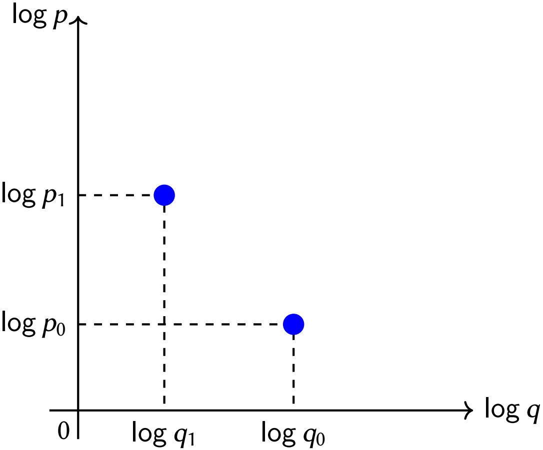
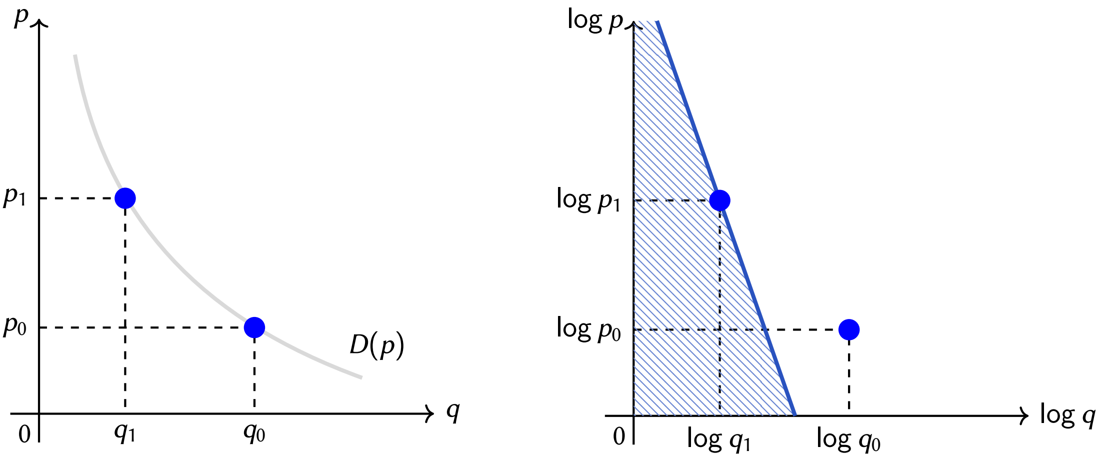


Geometric derivation of welfare bounds

Back



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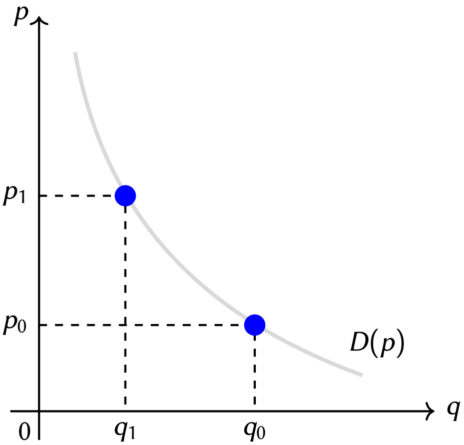

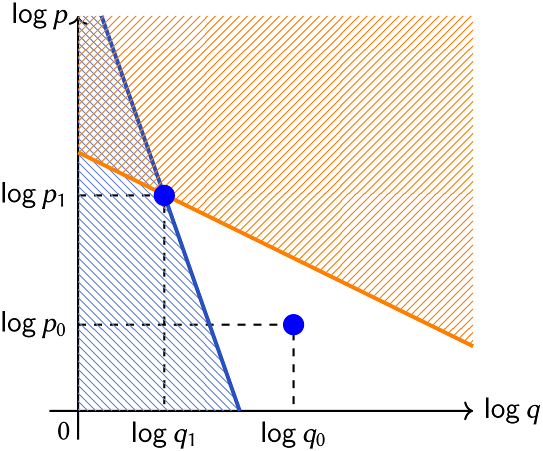

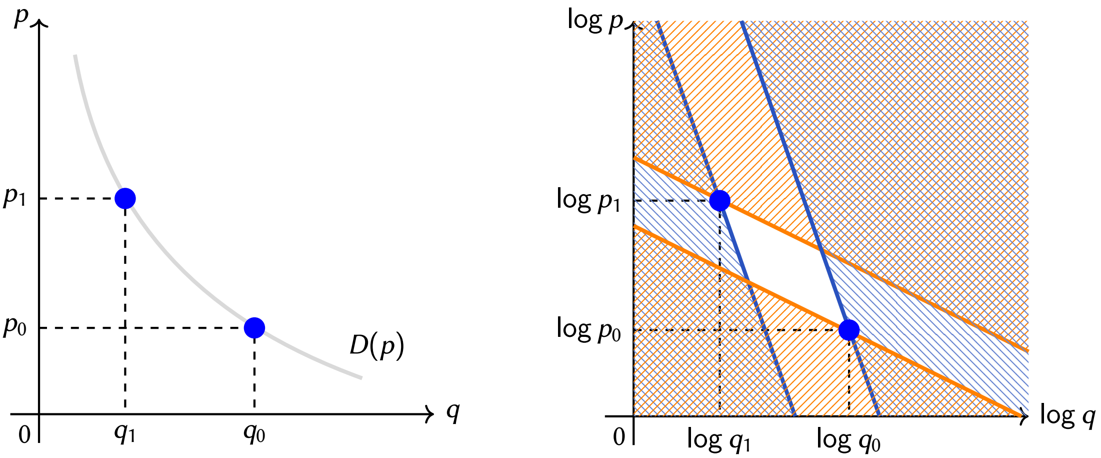

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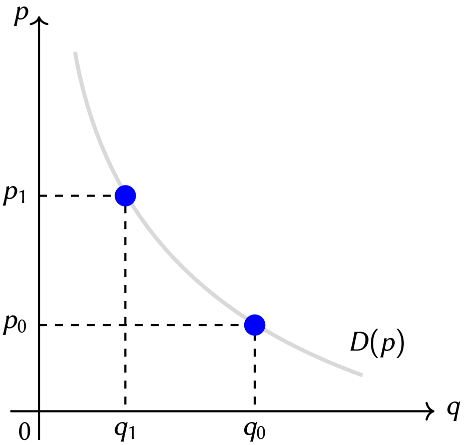
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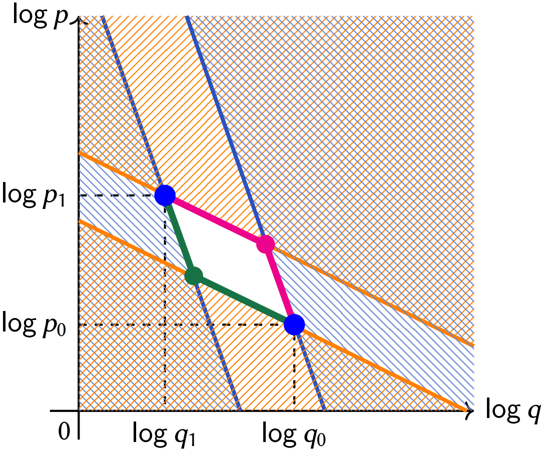
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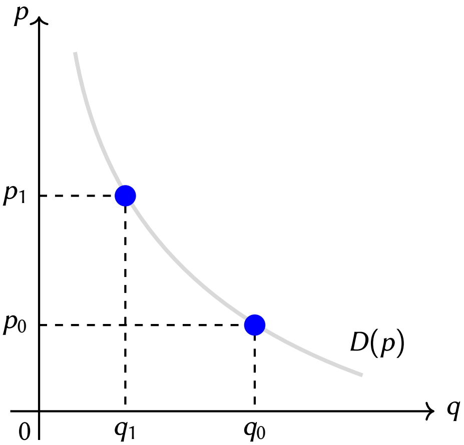

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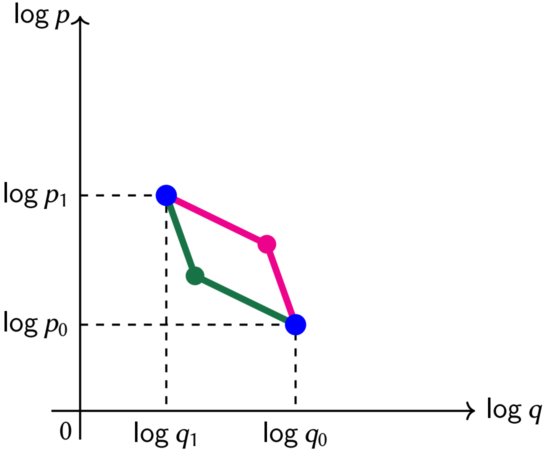

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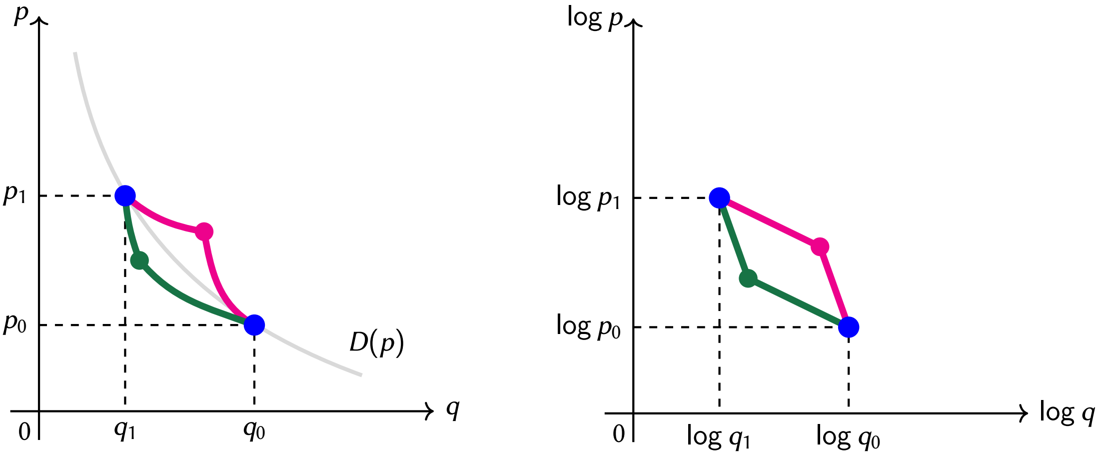
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Welfare bounds for basic model

Theorem 1 (welfare bounds).

The upper and lower bounds for the change in consumer surplus are attained by **2-piece CES interpolations**.

- These bounds can be easily computed.
- Tighter range of elasticities, $[\underline{\varepsilon}, \overline{\varepsilon}] \implies$ tighter bounds on consumer surplus.
- **Related literature:** "sufficient statistics" approach (Chetty, 2009; Kleven, 2021) maps from *local* elasticity estimates to *local* welfare estimates.

  $\rightsquigarrow$ Our approach maps from *global* elasticity bounds to *global* welfare bounds.

Choosing elasticity bands

- **Question.** What is a reasonable elasticity band?

  (a) Combine estimates from the literature.

  $\rightsquigarrow$ *E.g., "estimates of short run gasoline elasticities are between* -0.2 *and* -0.4*."*

  (b) Extrapolate from local estimates.

  $\rightsquigarrow$ *E.g., partial ID of treatment responses (Manski, 1997).*

  (c) Draw a (symmetric) band around the *average* elasticity.

$$\underline{\varepsilon} \leq \frac{\log q_1 - \log q_0}{\log p_1 - \log p_0} \leq \overline{\varepsilon}.$$

Discussion of basic model

Our welfare bounds for the basic model rely on a number of modeling choices:

1. Both points (p_0, q_0) and (p_1, q_1) on the demand curve are observed.

   In practice (e.g., counterfactuals), the analyst might observe p_0, p_1, and q_0, but not q_1.

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3. Only two points (p_0, q_0) and (p_1, q_1) on the demand curve are observed.

   In practice, the analyst might observe more points on the demand curve.

4. The points (p_0, q_0) and (p_1, q_1) on the demand curve are observed precisely.

   In practice, the analyst might be limited by sampling error.

Extensions to basic model

Our welfare bounds for the basic model rely on a number of modeling choices:

1. *In practice (e.g., counterfactuals), the analyst might observe p_0, p_1, and q_0, but not q_1.*

   $\Longrightarrow$ We show how to **extrapolate** from fewer observations.

2. *In practice, the analyst might make assumptions about demand curvature.*

   $\Longrightarrow$ We show how **demand curvature** assumptions lead to tighter bounds.

3. *In practice, the analyst might observe more points on the demand curve.*

   $\Longrightarrow$ We show how to **interpolate** with more observations.

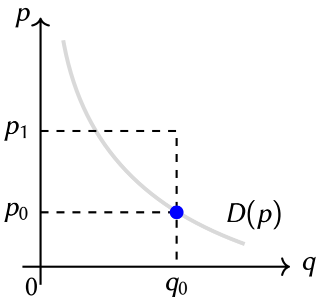
4. *In practice, the analyst might be limited by sampling error.*

   $\Longrightarrow$ We show how to incorporate **sampling error** into welfare bounds.

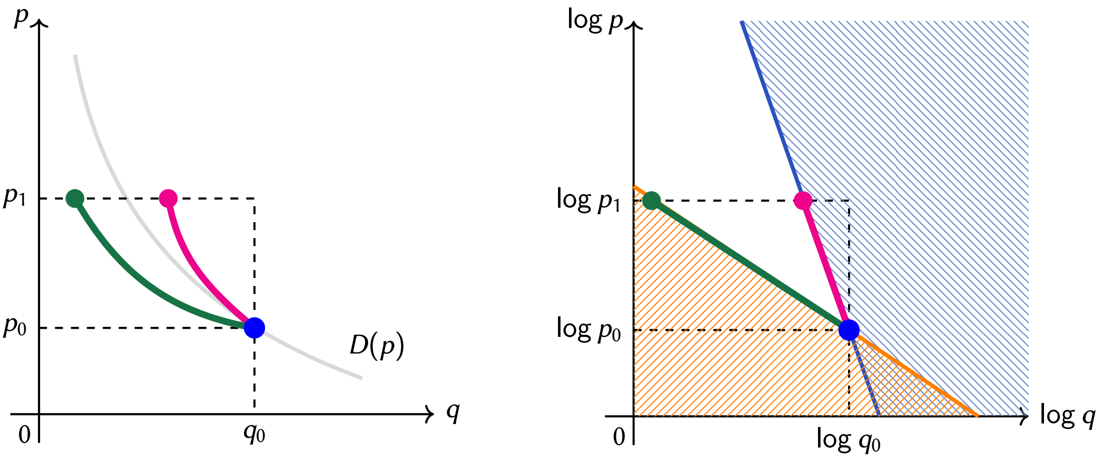
An analyst observes **1 point** on a demand curve: (p_0, q_0); p_1 is given.

We assume that elasticities between p_0 and p_1 lie in the interval $[\underline{\varepsilon}, \overline{\varepsilon}] \subset \mathbb{R}_{\leq 0}$.

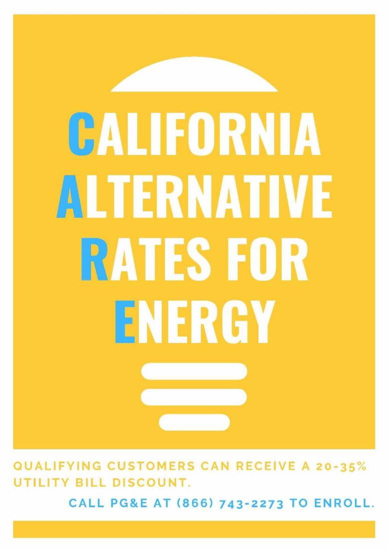
Question. What is the change in consumer surplus from p_0 to p_1?



② Extrapolating from less data: geometric intuition



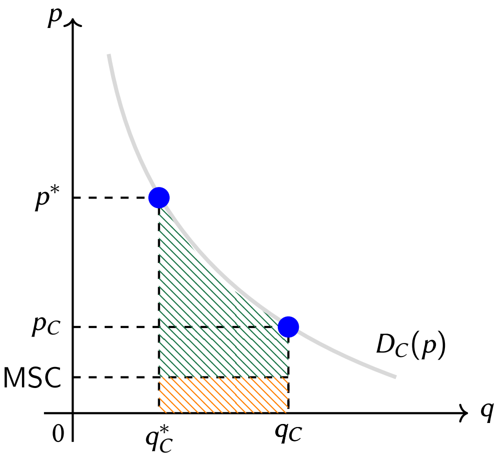
What is the welfare impact of CARE gas subsidies?

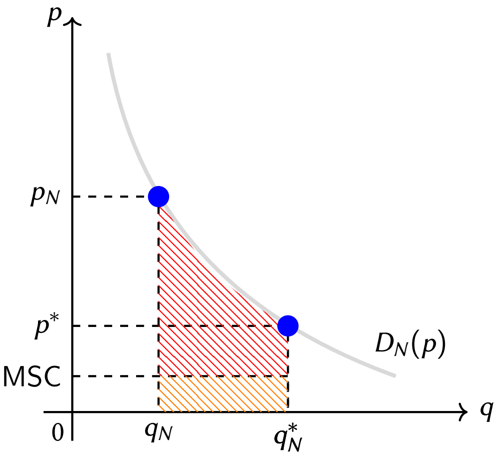


CARE Program:

- **Low income:** 20% discount on gas
  - $\rightsquigarrow$ Gas usage $\uparrow$
  - $\rightsquigarrow$ Consumer surplus $\uparrow$
  - $\rightsquigarrow$ Climate impact $\downarrow$
- **Other households:** gas price $\uparrow$ (given a fixed budget)
  - $\rightsquigarrow$ Gas usage $\downarrow$
  - $\rightsquigarrow$ Consumer surplus $\downarrow$
  - $\rightsquigarrow$ Climate impact $\uparrow$
- **Administrative cost:** $7M

Bounding counterfactual welfare from uniform pricing





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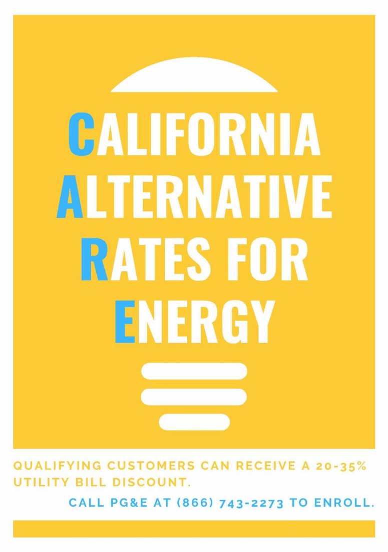

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Question: Is CARE net welfare improving?

Welfare impact of energy subsidies (Hahn and Metcalfe, 2021)

- **Empirical strategy:**
  - Randomly nudge eligible households to sign up for CARE.
  - Compute LATE based on gas usage with and without CARE (using nudges as an IV).
  - Interpret the LATE as an elasticity:

  $\rightsquigarrow$ *How much does gas usage change given a 20% discount in unit price?*

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  - Interpret the LATE as an elasticity:

  ⇝ *How much does gas usage change given a 20% discount in unit price?*
- **Modeling assumptions:**
  - The CARE program operates under a fixed budget.

  ⇝ The **counterfactual** "uniform" **price** is pinned down by observed quantities

$$N_n\left(P_n - P^*\right) Q_n = N_c\left(P^* - P_c\right) Q_c + A.$$

  - Consumer demand is linear.

Welfare impact of energy subsidies (Hahn and Metcalfe, 2021)

- **Elasticity estimates:**
  - Estimated CARE elasticity of $-\mathbf{0.35}$.
  - Assume non-CARE elasticity is $-\mathbf{0.14}$ (Auffhammer and Rubin, 2018).
- **Welfare estimates:**

CARE:	$+\ \$5.3M$
Non-CARE:	$-\ \$3.1M$
Admin Costs:	$-\ \$7.0M$
Net:	$-\ \$4.8M$

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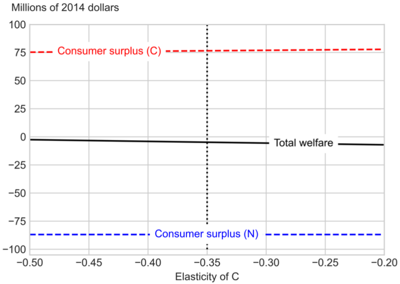
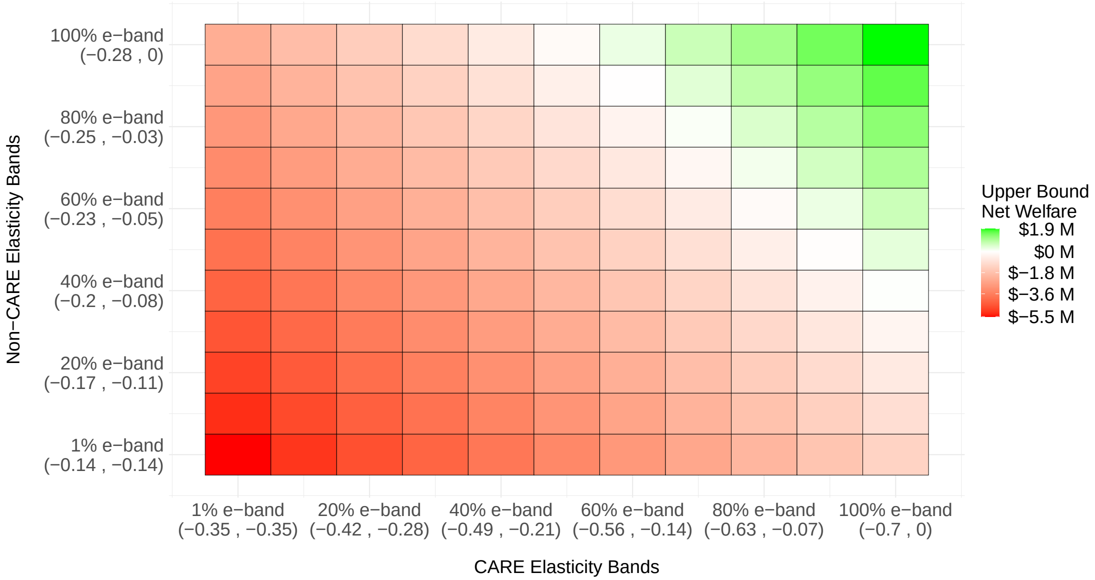


How robust is the negative welfare result?



Why might we expect the welfare results to flip?

- **#1.** Before imposing any assumptions, we can test the conservative (box) bounds.
  - $\rightsquigarrow$ They are positive! Something must give.
- **#2.** We "observe" p_1, q_1, ϵ_1 and p_0 but not q_0 or ϵ_0.
  - $\rightsquigarrow$ Our bounds account for uncertainty in both.
- **#3.** Our bounds are "adversarial".
  - $\rightsquigarrow$ They consider *all* feasible demand curves.
  - $\rightsquigarrow$ They default to joint uncertainty in ϵ_C and ϵ_N.

- **Why might we expect the welfare results to flip?**
  - **#1.** Before imposing any assumptions, we can test the conservative (box) bounds.
  - **#2.** We "observe" p_0, q_0, ε_0 and p_1 but not q_1 or ε_1.
  - **#3.** Our bounds are "adversarial."

- **So, how do we interpret these results?**
  - $\rightsquigarrow$ The Hahn and Metcalfe conclusion is pretty robust.
  - $\rightsquigarrow$ In fact, uncertainty in the non-CARE elasticity is not enough to break their result.

- **Why might we expect the welfare results to flip?**
  - **#1.** Before imposing any assumptions, we can test the conservative (box) bounds.
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  - **#3.** Our bounds are "adversarial."

- **So, how do we interpret these results?**
  - ⤳ The Hahn and Metcalfe conclusion is pretty robust.
  - ⤳ In fact, uncertainty in the non-CARE elasticity is not enough to break their result.
  - ⤳ But this might not be the case if the administrative cost had been lower… ▸ accounting

Our welfare bounds for the basic model rely on a number of modeling choices:

1. *In practice (e.g., counterfactuals), the analyst might observe p_0, p_1, and q_0, but not q_1.*

   $\Longrightarrow$ We show how to **extrapolate** from fewer observations.

2. *In practice, the analyst might make assumptions about demand curvature.*

   $\Longrightarrow$ We show how **demand curvature** assumptions lead to tighter bounds.

3. *In practice, the analyst might observe more points on the demand curve.*

   $\Longrightarrow$ We show how to **interpolate** with more observations.

4. *In practice, the analyst might be limited by sampling error.*

   $\Longrightarrow$ We show how to incorporate **sampling error** into welfare bounds.

"Notice that **these results depend on the fact** that the *PP* curve slopes upward, which in turn depends on the assumption that the **elasticity of demand falls with** c.

This assumption, which might alternatively be stated as an assumption that the elasticity of demand rises when the price of a good is increased, **seems plausible**.

In any case, it seems to be **necessary** if this model is to yield reasonable results, and I make the assumption without apology."

—Krugman (1979)

① Assumptions on demand curvature

Many models across different fields impose additional assumptions on demand:

(A1) Decreasing elasticity, or "Marshall's second law." (Marshall, 1890; Krugman, 1979)

(A2) Decreasing marginal revenue. (Myerson, 1981; Bulow and Roberts, 1989)

(A3) Log-concave demand. (Caplin and Nalebuff, 1991a; Bagnoli and Bergstrom, 2005)

(A4) Concave demand. (Rosen, 1965; Szidarovszky and Yakowitz, 1977; Caplin and Nalebuff, 1991a)

(A5) ρ-concave demand that generalizes **(A3)** and **(A4)**. (Caplin and Nalebuff, 1991a,b)

We call these "**concave-like** assumptions" on demand.

Many models across different fields impose additional assumptions on demand:

(A6) Convex demand. (Svizzero, 1997; Aguirre, Cowan and Vickers, 2010; Tsitsiklis and Xu, 2014)

(A7) Log-convex demand. (Caplin and Nalebuff, 1991b; Aguirre, Cowan and Vickers, 2010)

(A8) ρ-convex demand that generalizes **(A6)** and **(A7)**. (Caplin and Nalebuff, 1991a,b)

We call these "**convex-like** assumptions" on demand.

Relationships between curvature assumptions

Concave-like assumptions

(A1) Decreasing elasticity

(A2) Decreasing MR

(A3) Log-concave demand

(A4) Concave demand

(A5) ρ-concave demand

Convex-like assumptions

(A6) Convex demand

(A7) Log-convex demand

(A8) ρ-convex demand

$$\begin{tikzcd} & & (A1) \\ (A4) \arrow[r, Rightarrow] & (A3) \arrow[ur, Rightarrow] \arrow[dr, Rightarrow] & \\ & & (A2) \end{tikzcd}$$

$$(A7) \Longrightarrow (A6).$$

① Assumptions on demand curvature: welfare bounds

Theorem 2a. (concave-like assumptions).

The **lower** bound for the change in consumer surplus are attained by:

(A1) **decreasing elasticity:** a *CES* interpolation; $D(p) = \theta_1 p^{-\theta_2}$

(A2) **decreasing MR:** a *constant MR* interpolation; $D(p) = \theta_1 (p - \theta_2)^{-1}$

(A3) **log-concave demand:** an *exponential* interpolation; $D(p) = \theta_1 e^{-\theta_2 p}$

(A4) **concave demand:** a *linear* interpolation; $D(p) = \theta_1 - \theta_2 p$

(A5) **ρ-concave demand:** a ρ-*linear* interpolation. $D(p) = [1 + \rho(\theta_1 - \theta_2 p)]^{1/\rho}$

Theorem 2b. (convex-like assumptions).

The **upper** bound for the change in consumer surplus are attained by:

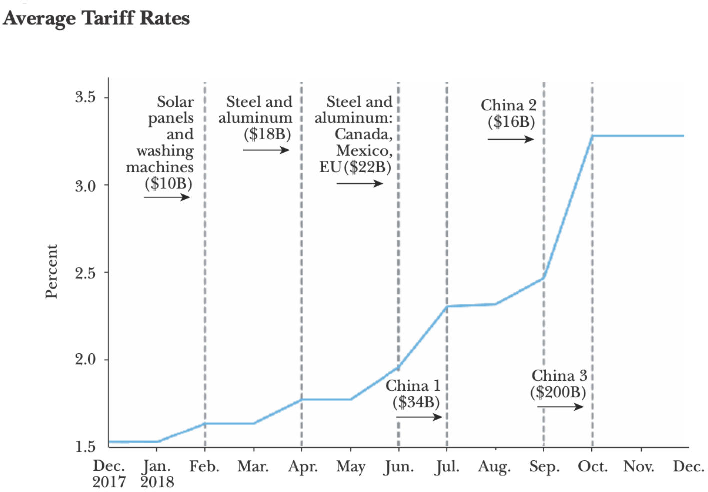
(A6) **convex demand:** a *linear* interpolation; $D(p) = \theta_1 - \theta_2 p$

(A7) **log-convex demand:** an *exponential* interpolation; $D(p) = \theta_1 e^{-\theta_2 p}$

(A8) **ρ-convex demand:** a ρ-*linear* interpolation. $D(p) = [1 + \rho(\theta_1 - \theta_2 p)]^{1/\rho}$

▸ Geometric intuition

Example: evaluating the deadweight loss of the Trump tariffs



Source: *Amiti, Redding and Weinstein (2019)*

Example: evaluating the deadweight loss of the Trump tariffs

How Many Tariff Studies Are Enough?

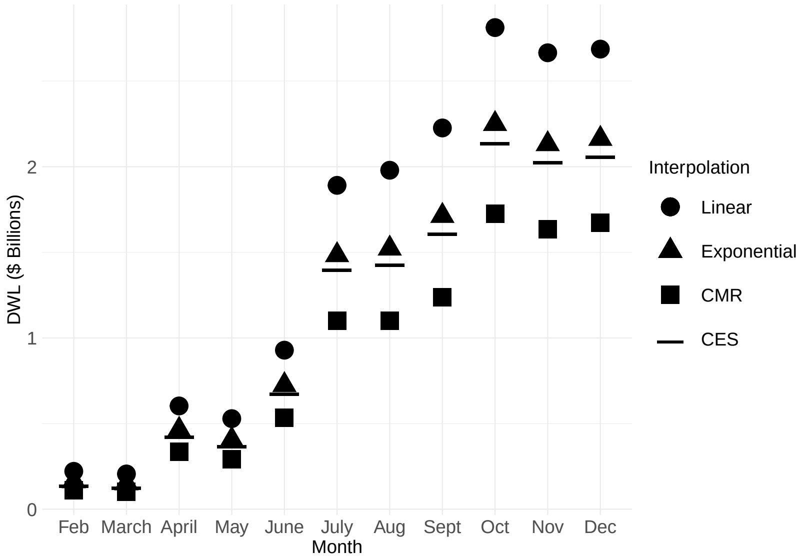
The trade war hits consumers and exports, two more papers say.

By The Editorial Board
Jan. 20, 2020 4:39 pm ET

PRINT TEXT 169

Source: WSJ Editorial Board

Bounding the tariff DWL across countries and products



Theorem 2a. (concave-like assumptions).

The **lower** bound for the change in consumer surplus are attained by:

(A1) decreasing elasticity: a *CES* interpolation. $D(p) = \theta_1 p^{-\theta_2}$

Variable change:

$$\eta(\pi) = -\frac{e^{\pi} D'(e^{\pi})}{D(e^{\pi})} \quad \text{where } \pi = \log p \implies D(p) = q_0 \exp\left[-\int_{\log p_0}^{\log p} \eta(\pi)\, \mathrm{d}\pi\right].$$

Step #1: change of variables

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Constraint (on the mean of η)**:**

$$\mathcal{E} = \left\{\eta \text{ is increasing s.t. } \int_{\log p_0}^{\log p_1} \eta(\pi)\, \mathrm{d}\pi = \log\left(\frac{q_0}{q_1}\right)\right\}.$$

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Welfare:

$$\begin{cases} \overline{\Delta \mathrm{CS}} = q_0 \cdot \max\limits_{\eta \in \mathcal{E}} \displaystyle\int_{p_0}^{p_1} \exp\left[-\int_{\log p_0}^{\log p} \eta(\pi)\, \mathrm{d}\pi\right] \mathrm{d}p, \\ \underline{\Delta \mathrm{CS}} = q_0 \cdot \min\limits_{\eta \in \mathcal{E}} \displaystyle\int_{p_0}^{p_1} \exp\left[-\int_{\log p_0}^{\log p} \eta(\pi)\, \mathrm{d}\pi\right] \mathrm{d}p. \end{cases}$$

Step #2: establishing a partial order

Definition: $\eta_1 \succeq \eta_2$ if η_1 is a mean-preserving spread of η_2, *i.e.*,

$$\eta_1 \succeq \eta_2 \iff \int_{\log p_0}^{\log p} \eta_1(\pi)\, \mathrm{d}\pi \geq \int_{\log p_0}^{\log p} \eta_2(\pi)\, \mathrm{d}\pi \qquad \forall\, p \in [p_0, p_1].$$

- This defines a *partial order* on $\mathcal{E}$.
  - $\Rightarrow$ Can think of this as second-order stochastic dominance.
  - $\Rightarrow$ Because η is increasing, can think of η as a CDF (shifted and scaled).

Step #2: connecting to welfare

Lemma: The welfare objective is decreasing in the partial order $\succeq$:

$$\eta_1 \succeq \eta_2 \implies \int_{p_0}^{p_1} \exp\left[-\int_{\log p_0}^{\log p} \eta_1(\pi)\,\mathrm{d}\pi\right]\,\mathrm{d}p \leq \int_{p_0}^{p_1} \exp\left[-\int_{\log p_0}^{\log p} \eta_2(\pi)\,\mathrm{d}\pi\right]\,\mathrm{d}p.$$

Proof: Pointwise comparison of the integrands.

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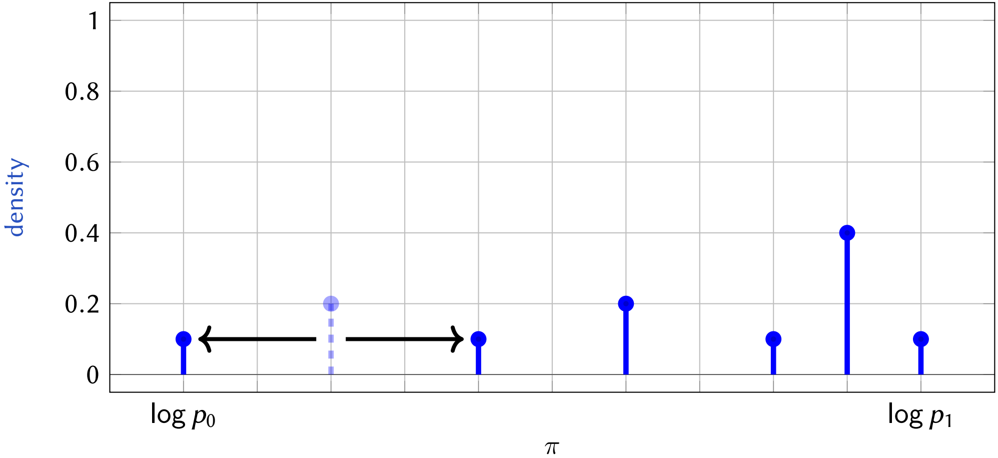
Proof: Pointwise comparison of the integrands.

Corollary. The lower (*resp.*, upper) bound is attained by iteratively applying mean-preserving spreads (*resp.*, mean-preserving contractions) to $\eta(\pi)$.

Step #3: deriving the *lower* bound

Consider the density that generates $\eta(\pi)$, where $\eta(\pi)$ is viewed as a CDF:



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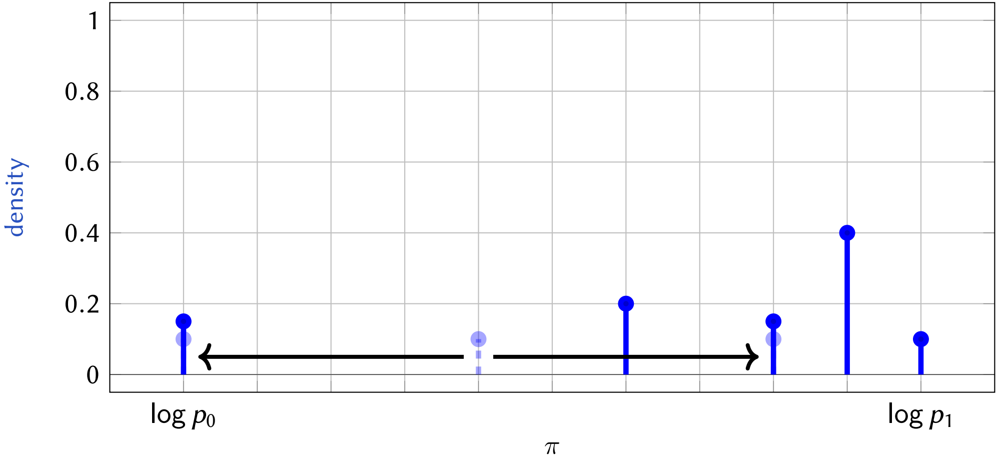

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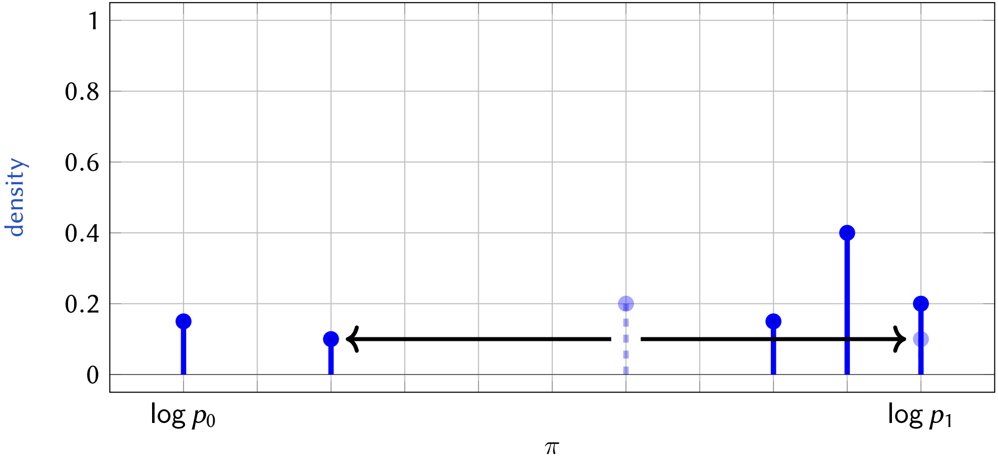

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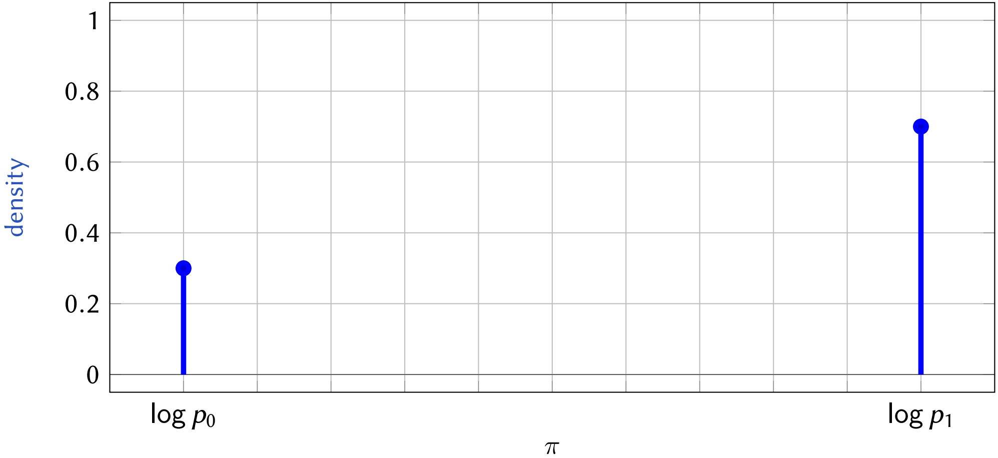
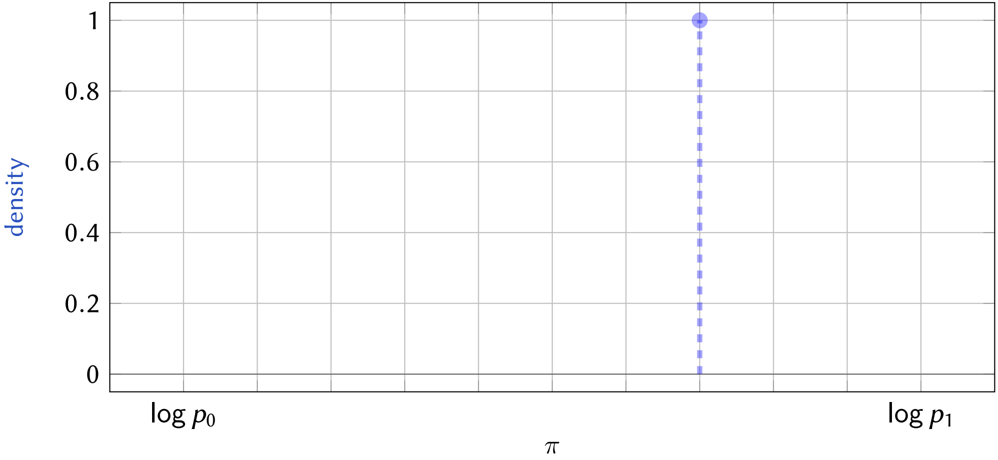


Step #3: deriving the *lower* bound

So the $\eta(\pi)$ that attains the **lower bound on welfare** is **constant** between p_0 and p_1:

Similarly, the $\eta(\pi)$ that attains the **upper bound on welfare** is a **step function**.



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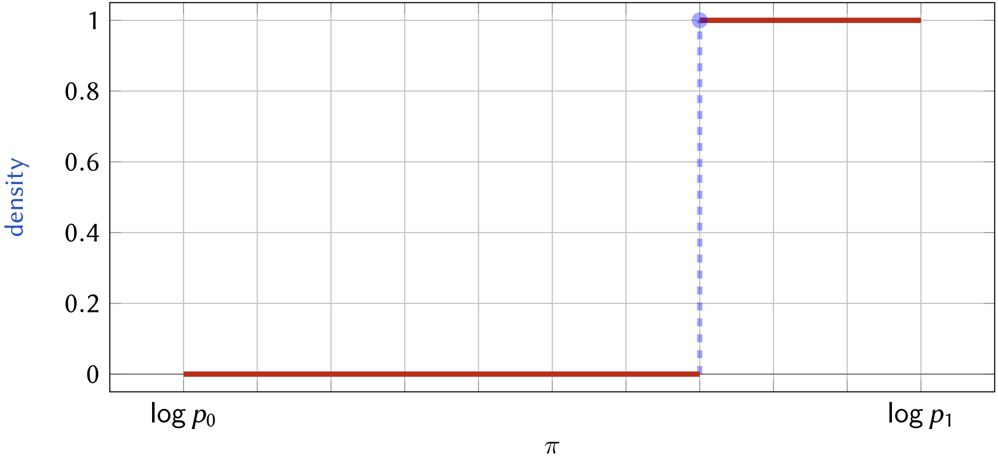

Step #3: deriving welfare bounds

- Mapping back from $\eta(\pi)$ into demand curves $D(p)$:

$$\eta(\pi) \text{ is constant} \iff D(p) \text{ has constant elasticity.}$$

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- This proves the bounds for assumption (A1) (decreasing elasticity):
  - The **upper bound** is attained by a 2-piece CES interpolation.
  - The **lower bound** is attained by a 1-piece CES interpolation.

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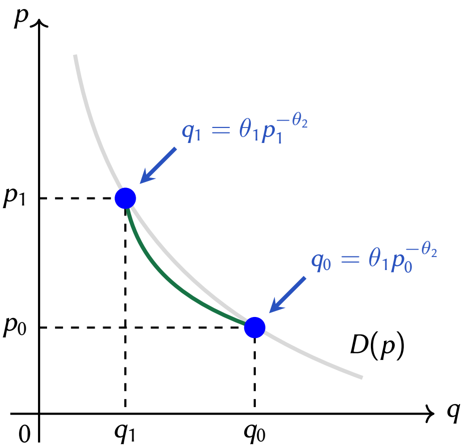
- This proves the bounds for assumption (A1) (decreasing elasticity):
  - The **upper bound** is attained by a 2-piece CES interpolation.
  - The **lower bound** is attained by a 1-piece CES interpolation.
- The same proof strategy works for all the other assumptions.



- We solve simultaneously:

$$\begin{cases} q_0 &= \theta_1 p_0^{-\theta_2}, \\ q_1 &= \theta_1 p_1^{-\theta_2}. \end{cases}$$

The solution (θ_1^*, θ_2^*) determines the interpolation:

$$D(p) = \theta_1^* p^{-\theta_2^*}.$$

- This can be done for each assumption, as each curve has 2 parameters.

Theorem 2a. (concave-like assumptions).

The **lower** bound for the change in consumer surplus are attained by:

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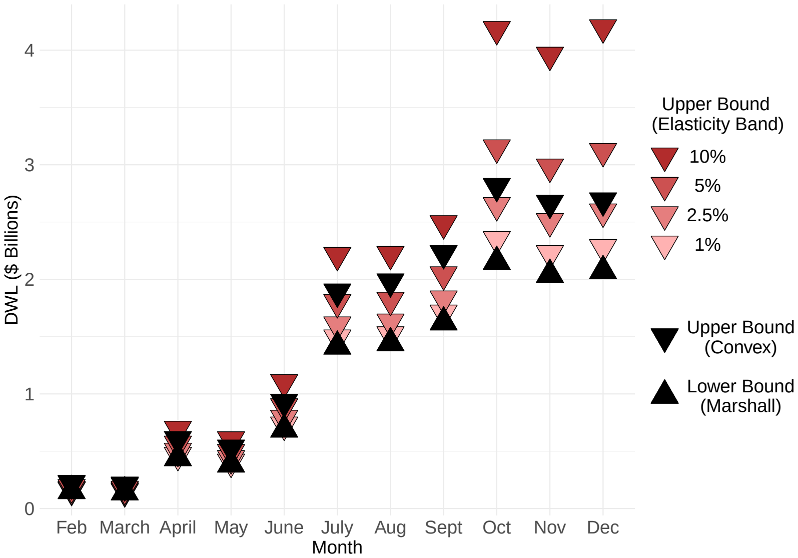
- However, we can
  - **#1.** combine different demand curvature assumptions; or
  - **#2.** combine demand curvature assumptions with assumption that elasticity lies in $[\underline{\varepsilon}, \overline{\varepsilon}]$.

Bounding the tariff DWL across countries and products



Interpretation of tariff DWL bounds

- Our **lower bound** on DWL incurred over 2018 is **$12.6 billion**.
  - The tariff revenue gained over 2018 is $15.6 billion.
  - A linear interpolation yields a DWL estimate of $16.8 billion.
- **Question.** Is there a sense in which $16.8 billion might be an **overestimate**?
  - Yes, if we expect the change in elasticity down the demand curve to be small.
  - $\rightsquigarrow$ If we expect the demand curve to be **convex**, then $16.8 billion is an **upper bound**.
- **Question.** Is there a sense in which $16.8 billion might be an **underestimate**?
  - Yes, if we expect the change in elasticity down the demand curve to be large.

Extensions to the basic model

Our welfare bounds for the basic model rely on a number of modeling choices:

1. *In practice, the analyst might make assumptions about demand curvature.*

   $\Longrightarrow$ We show how **demand curvature** assumptions lead to tighter bounds.

2. *In practice (e.g., counterfactuals), the analyst might observe p_0, p_1, and q_1, but not q_0.*

   $\Longrightarrow$ We show how to **extrapolate** from fewer observations.

3. *In practice, the analyst might observe more points on the demand curve.*

   $\Longrightarrow$ We show how to **interpolate** with more observations. ▸ Details

4. *In practice, the analyst might be limited by sampling error.*

   $\Longrightarrow$ We show how to incorporate **sampling error** into welfare bounds. ▸ Details

Further extensions: welfare beyond ΔCS

#1. Producer surplus works just as well as CS.

#2. Can handle heterogeneity + distributional questions.

#3. Can handle alternative welfare measures like EV and CV.

#4. Can handle multiple objectives at once.

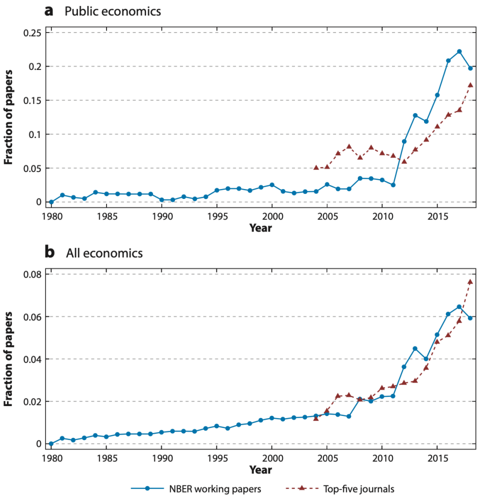
- $\rightsquigarrow$ E.g., Pareto-weighted consumer surplus + DWL.

#5. Can handle multi-product markets.

- $\rightsquigarrow$ At least under constraints on cross-price and own-price elasticities.

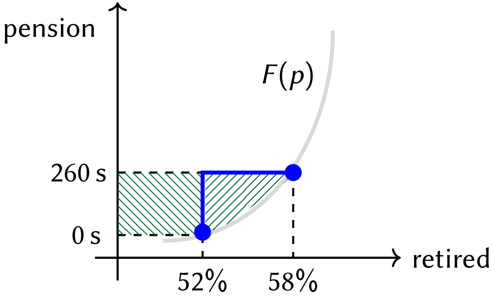
Skip to the end

MVPF and the "sufficient statistics" approach

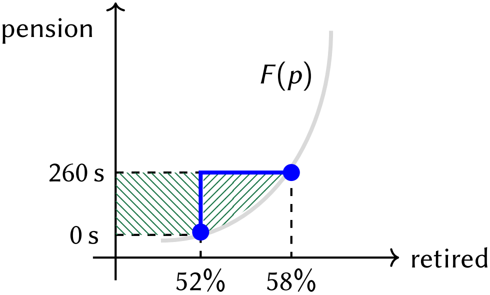


Source: Kleven (2021)

MVPF example: WTP of 1911 UK pension recipients



- Based on Giesecke and Jäger (2021).
- **Setting:** pensions created for poor >70-year-olds in the UK in 1911.
- **Question:** what is the MVPF of the pension policy?
- **Approach:** MVPF $=$ (WTP for not working) / (cost of pension).
- **Method:** compute % marginal workers via RD; assume marginal workers' WTP $= 0$.



- What is a "demand curve" here?
- **Problem #1:** we don't actually know the distribution of incomes.
- **Problem #2:** the inherent cost/value of retirement might be heterogeneous.
- **Approach:** each retirement is a discrete choice: i retires iff $p \geq w_i$. $w_i \overset{iid}{\sim} F$, where $F(p) =$ prob of retirement.
- **Model:** $\Delta W = \int_{p_0}^{p_1} F(p)\, \mathrm{d}p$.

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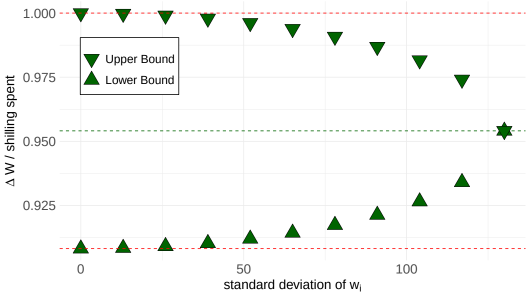

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- **Problem #2:** the inherent cost/value of retirement might be heterogeneous.
- **Approach:** Each retirement is a discrete choice: i retires iff $p \geq w_i$. Model uncertainty in the variance of the prob of retirement $F(p)$.
- **Model:** $\Delta W = \int_{p_0}^{p_1} F(p)\,\mathrm{d}p$.

Sufficient statistics for income taxation

- Consider an exogenous change in marginal tax rates.
- Estimate a *local elasticity* of taxable income.
- Invoke envelope theorem to argue other effects are 2nd order.
- Compute the marginal change in welfare as a function of measured elasticity

Feldstein (1999): $$\frac{\mathrm{d}\,W(\tau)}{\mathrm{d}\tau} = \tau \cdot \frac{\mathrm{d}\,\mathsf{TI}(\tau)}{\mathrm{d}\tau}.$$

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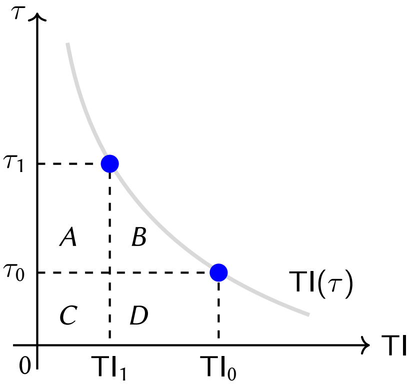
$$\text{Feldstein (1999)}: \quad \frac{\mathrm{d}\,W(\tau)}{\mathrm{d}\tau} = \tau \cdot \frac{\mathrm{d}\,\mathsf{TI}(\tau)}{\mathrm{d}\tau}.$$

- To obtain total welfare change, integrate $\mathrm{d}\,W(\tau)/\mathrm{d}\tau$.

- The change in welfare:

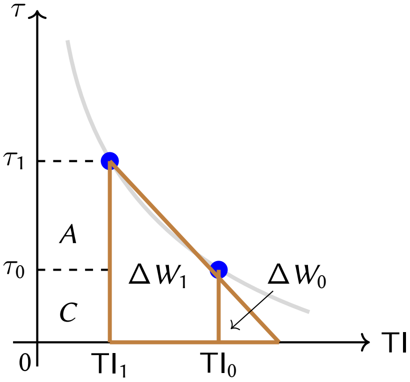
$$
\begin{aligned}
\Delta W &= W(\tau_1) - W(\tau_0) \\
&= \int_{\tau_0}^{\tau_1} \tau \cdot \mathrm{TI}'(\tau)\, \mathrm{d}\tau \\
&= [\tau_1\, \mathrm{TI}(\tau_1) - \tau_0\, \mathrm{TI}(\tau_0)] - \int_{\tau_0}^{\tau_1} \mathrm{TI}(\tau)\, \mathrm{d}\tau \\
&= -\left(\text{area } B + \text{area } D\right).
\end{aligned}
$$



A robust bounds approach to Feldstein (1999)

- The change in welfare (Feldstein, 1999):

$$\Delta W \approx \Delta W_1 - \Delta W_0.$$



Elasticity estimates and welfare: Feldstein (1995/9)

- **Data**: the Tax Reform Act of 1986 dramatically reduced top tax rates.
- **Estimates:** Feldstein “diff-in-diff" estimates range from -1.04 to -1.48.
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- **Illustrative example:** consider a taxpayer with \$180,000 of taxable income.
  - A linear interpolation predicts DWL of \$7,458.
- **Robust bounds for the example:**
  - Box bounds for the DWL are \$6,615 and \$8,301.
  - Elasticity bounds using $[-1.33, -0.55]$ are \$7,400 and \$7,418.

  $\rightsquigarrow$ The elasticity bounds reject the linear interpolation!

Summing up

- **This paper.** Develops a framework to bound welfare based on economic reasoning.
- **Building on previous work.** Hope to make the case that everyone should use this.
- **Use cases.** Draw/assess conclusions from empirical objects commonly estimated.
- **Future work.** We're excited about this.
  - Robustness for structural IO-style problems (e.g., inference with endogenous pricing, merger screens, welfare in horizontally differentiated good markets).
  - Robustness for new goods and price indices (e.g., the CPI).
  - Robustness for larger macro models (e.g., extending ACR, ACDR).

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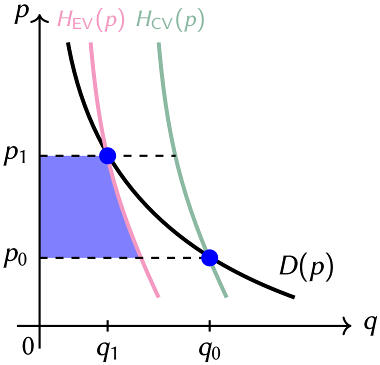
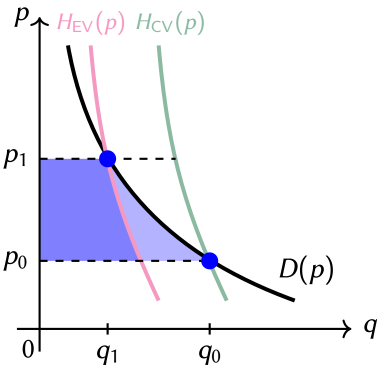
Consumer surplus provides bounds for equivalent and compensating variations.



- **Generally:** EV $\leq$ CS $\leq$ CV.

Consumer surplus provides bounds for equivalent and compensating variations.



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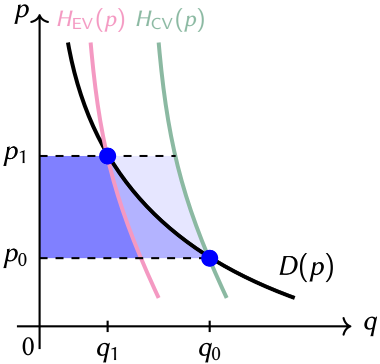
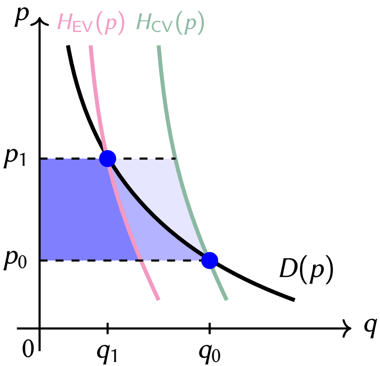


- **Generally:** EV $\leq$ CS $\leq$ CV.

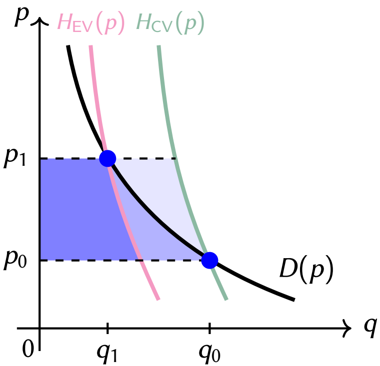
Mapping CS to EV/CV when income effects are small

Consumer surplus provides bounds for equivalent and compensating variations.



- **Generally:** EV $\leq$ CS $\leq$ CV.
- When income effects are 0 (e.g., with quasilinearity): EV $=$ CS $=$ CV.
- When income effects are ≈ 0:
  EV $\approx$ CS $\approx$ CV (Willig, 1976)
  (also if demand is pretty inelastic).

We can compute EV/CV bounds under assumptions about the *Hicksian* demand curve.



- **But!** we don't observe counterfactual expenditures.
- Need to bound $e(p_1, u_0)$ for CV.
- Need to bound $e(p_0, u_1)$ for EV.
- This maps to our "1-point" extension.

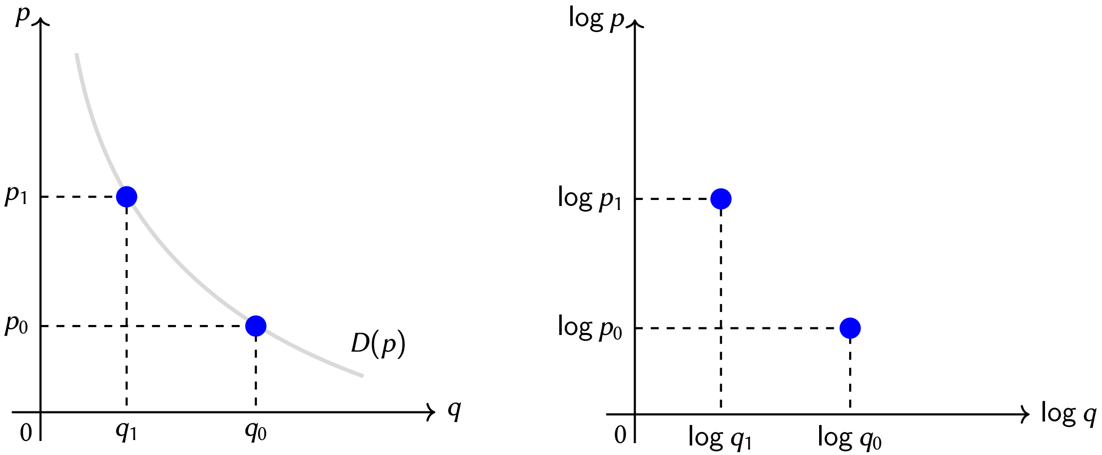
◂ Basic Model ▸ Skip to End

Theorem 2a. (concave-like assumptions).

The **lower** bound for the change in consumer surplus are attained by:

(A1) decreasing elasticity: a *CES* interpolation. $D(p) = \theta_1 p^{-\theta_2}$

Marshall's second law (decreasing elasticity) $\iff$ $\log q$ is concave in $\log p$.



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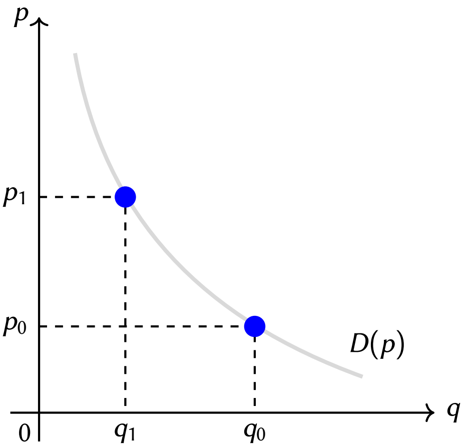

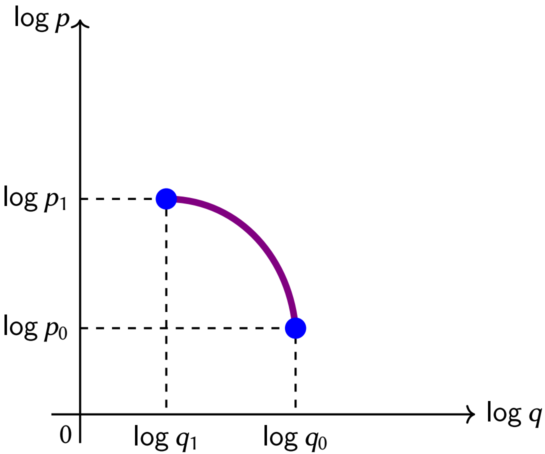

Assumptions on demand curvature: geometric intuition

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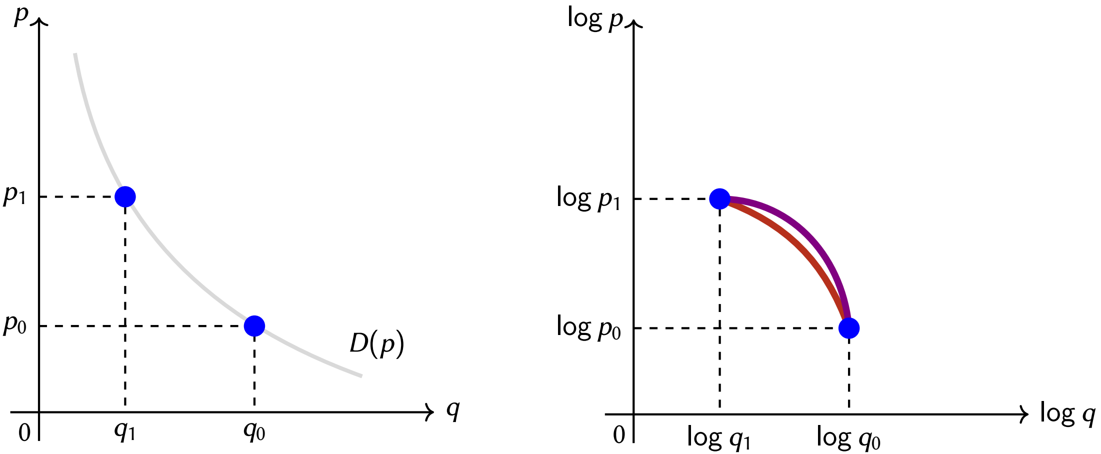

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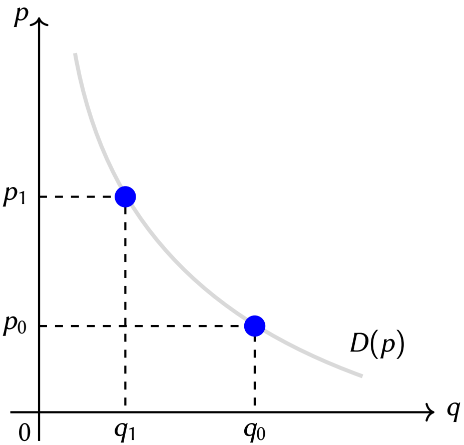

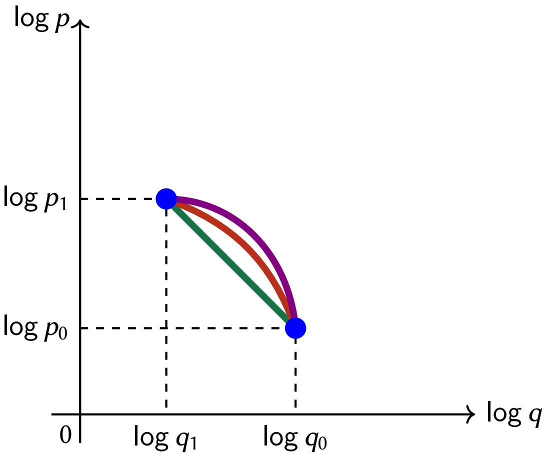

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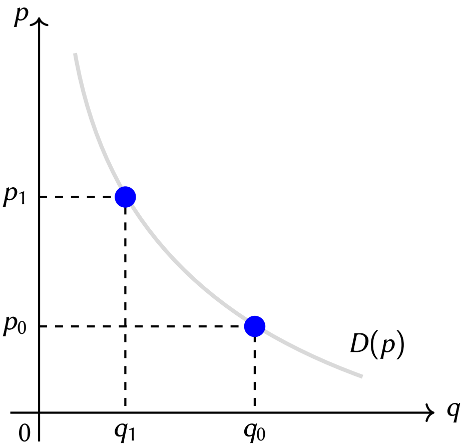

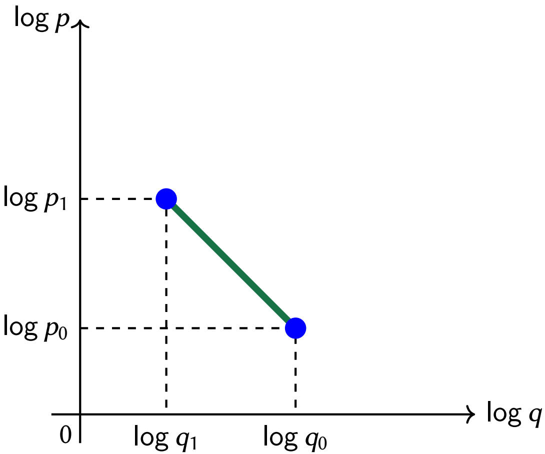

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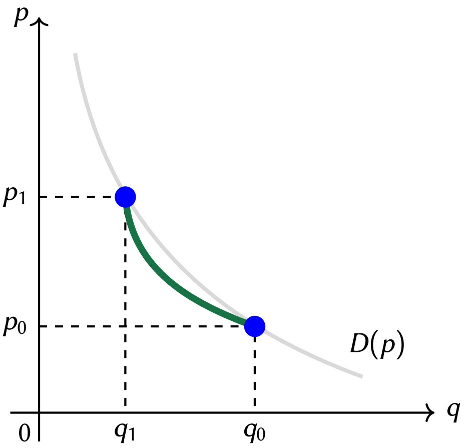
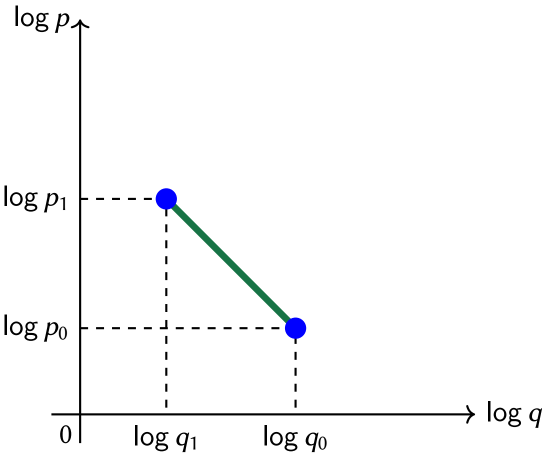
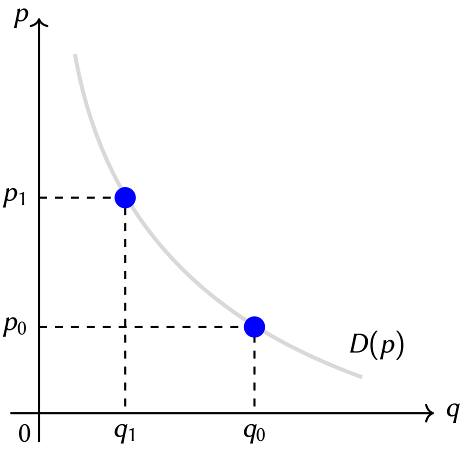
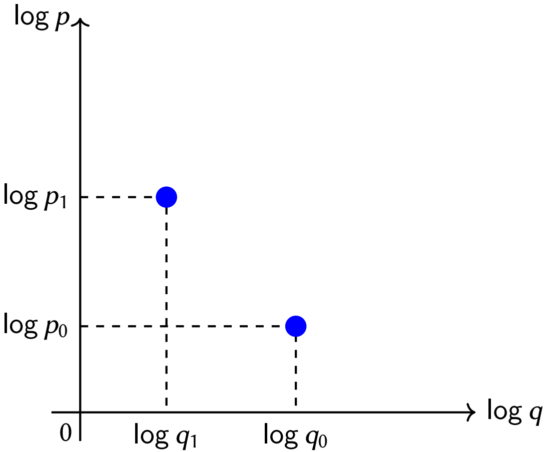




Back

Marshall's second law (decreasing elasticity) + elasticity lies in $[\underline{\varepsilon}, \overline{\varepsilon}]$.





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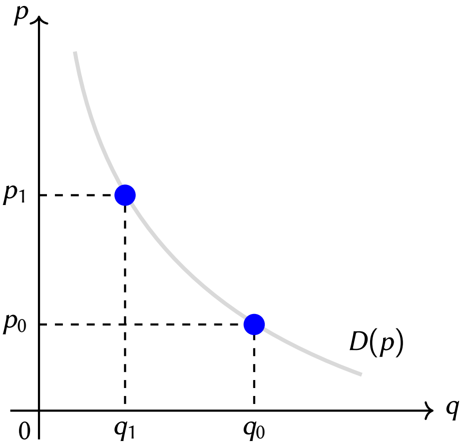

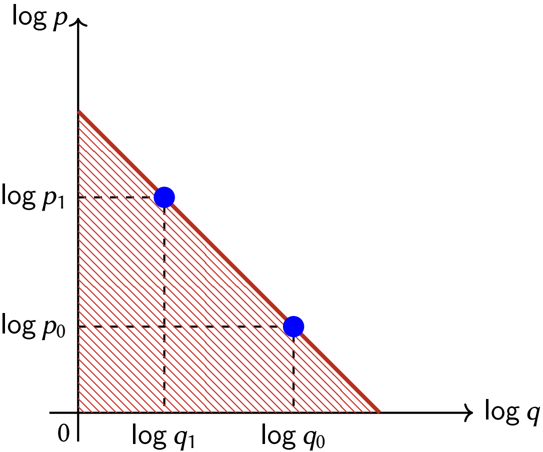

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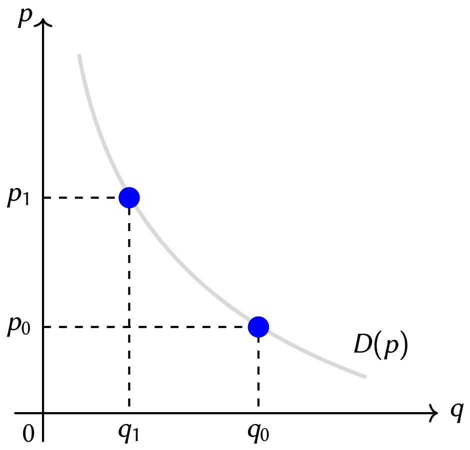

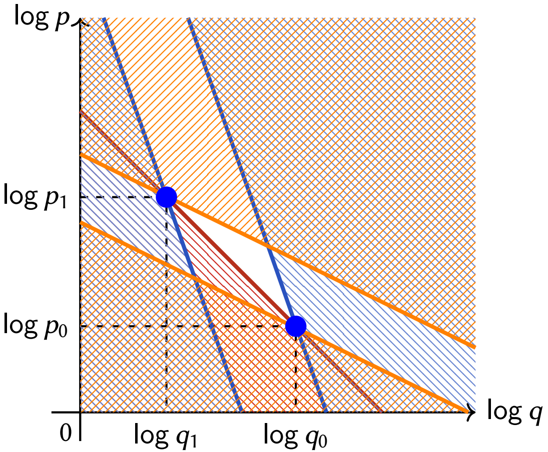

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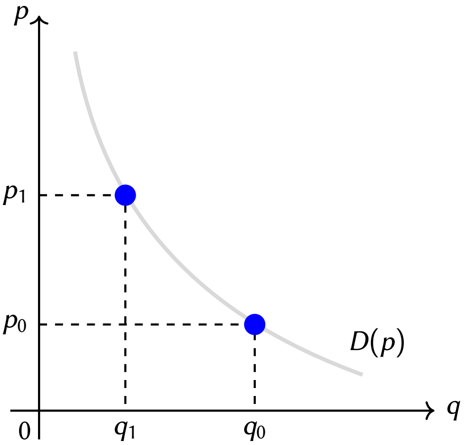

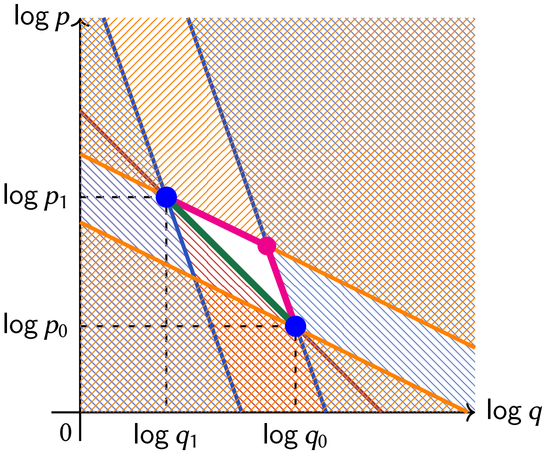

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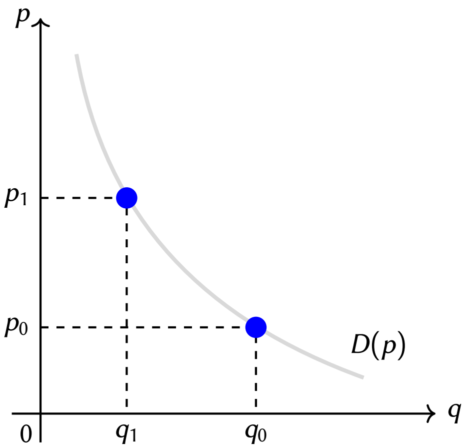

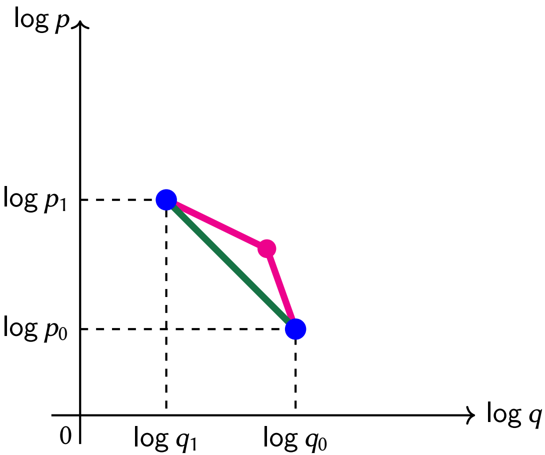

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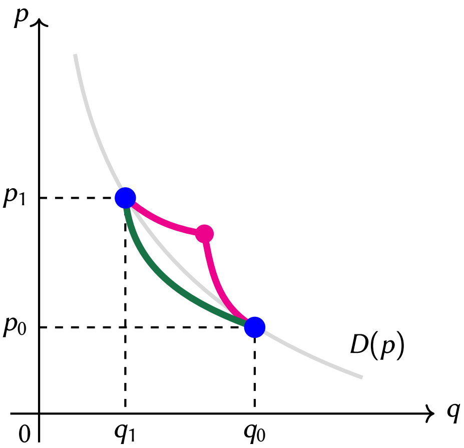
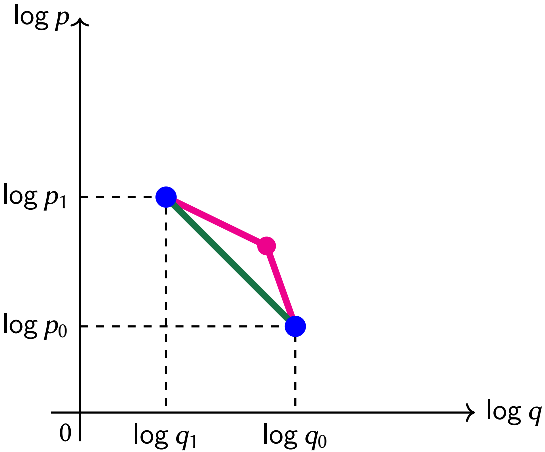
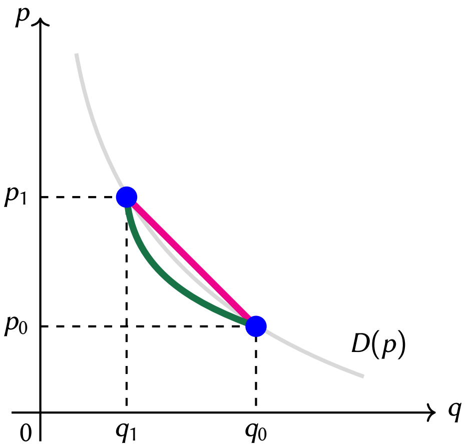




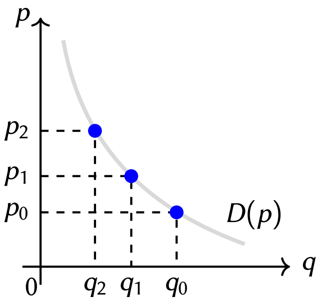
Marshall's second law (decreasing elasticity) + convex demand.

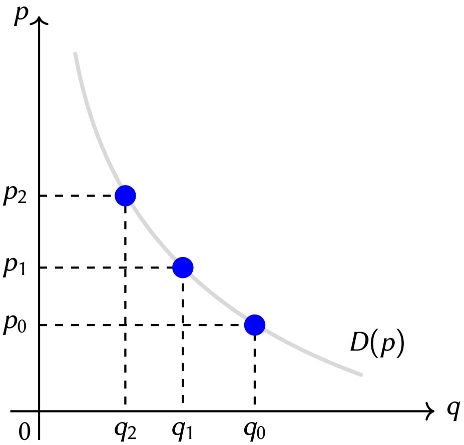


An analyst observes **3 points** on a demand curve: (p_0, q_0), (p_1, q_1), and (p_2, q_2).

We assume that elasticity between p_0 and p_2 lie in the interval $[\underline{\varepsilon}, \overline{\varepsilon}] \subset \mathbb{R}_{\leq 0}$.

Question. What is the change in consumer surplus from p_0 to p_2?

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q_2
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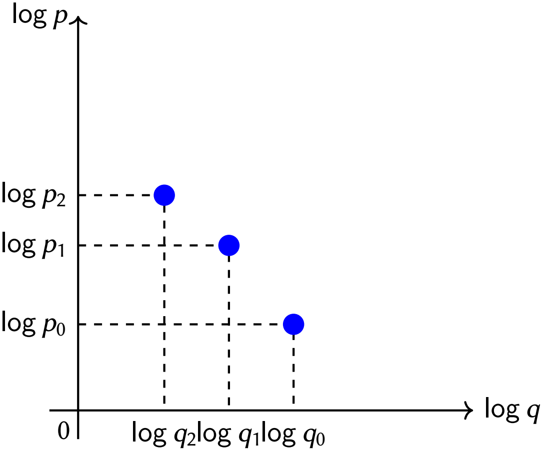
q_0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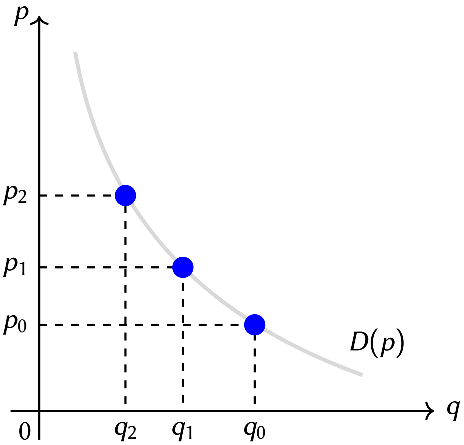
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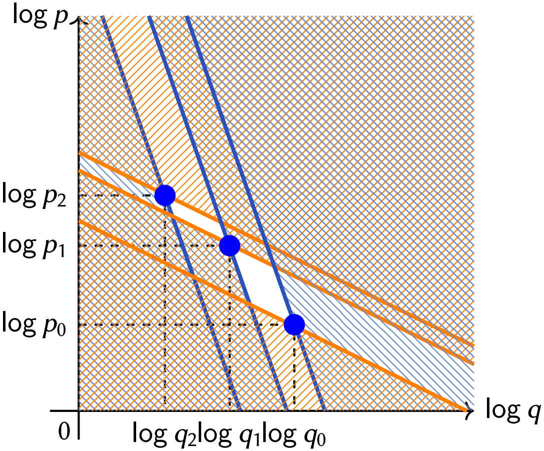
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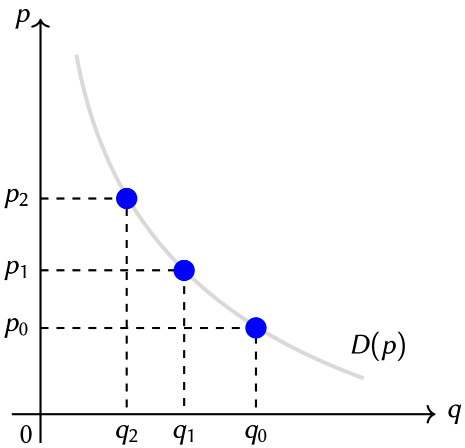
$\log q$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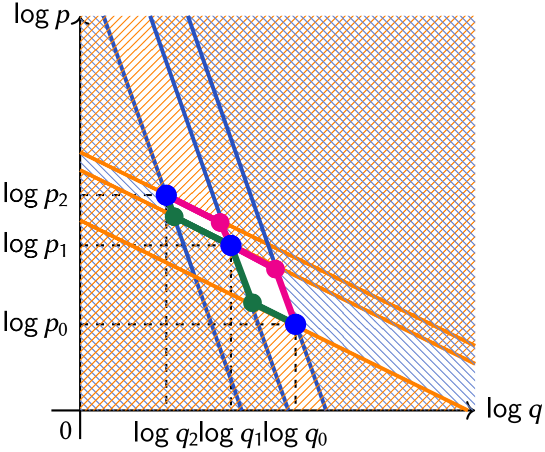
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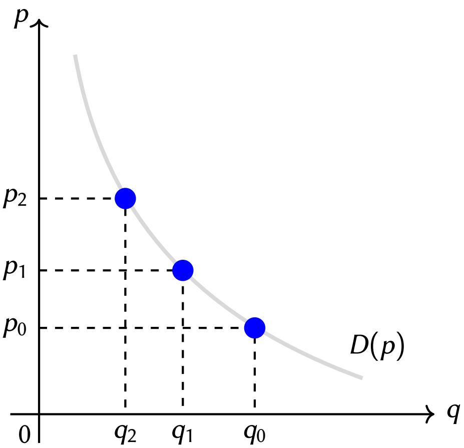
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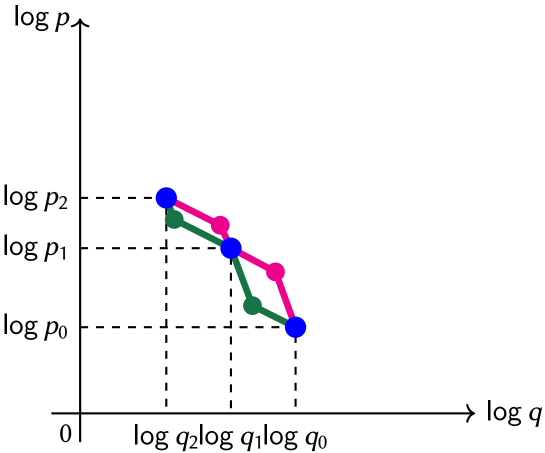
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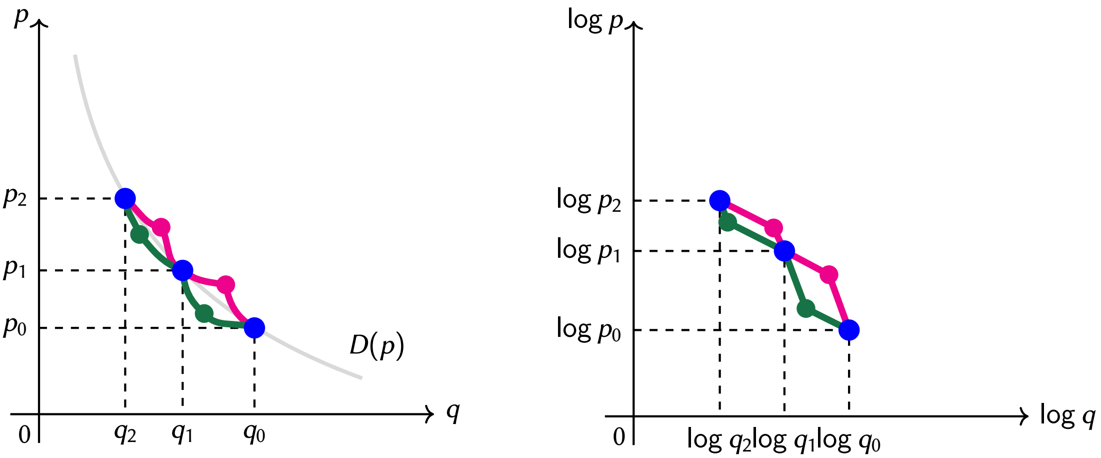

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q_2
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log p_0
log q
0
log $q_2$log $q_1$log q_0

Quantities demanded might be noisily observed:

$$q_1 = D(p_1) + e \quad \text{where} \quad e \sim \mathcal{N}\left(0, \sigma^2/N_1\right).$$

Question. What is the 95% CI on the change in consumer surplus from p_0 to p_1?

$\Longrightarrow$ The bounds $\overline{\Delta \mathrm{CS}}(q_0, q_1)$ and $\underline{\Delta \mathrm{CS}}(q_0, q_1)$ are monotonic in q_1.

$\Longrightarrow$ CIs on ΔCS can be obtained by plugging in the CIs of q_1.